# Unicast, Multicast, and Broadcast in Enhanced Fibonacci Cubes

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## Abstract

*The enhanced Fibonacci cube (EFC) is defined based on the sequence $F_n = 2F_{n-2} + 2F_{n-4}$. It contains the Fibonacci cube as a subgraph and maintains virtually all the desirable properties of the Fibonacci cube, and it also possesses properties such as the Hamiltonian property that the Fibonacci cube does not have. In this paper, we study data routing, namely, unicast, multicast and broadcast in the enhanced Fibonacci cube. The time and traffic steps are used to measure the efficiency of routing algorithms. The unicast algorithm, which uses a Hamming distance path for any two nodes in EFC, is time and traffic optimal. The broadcast algorithm which employs the enhanced Fibonacci tree is traffic optimal and near time optimal. Two heuristic multicast algorithms are proposed which are based on an enhanced Fibonacci tree and a Hamiltonian cycle, respectively.*

**Index terms**: *broadcasting, communication, Fibonacci numbers, interconnection networks, multicasting, unicasting.*

## 1 Introduction

The hypercube is a powerful network that is able to perform various kinds of parallel computation and simulate many other networks. However, the number of nodes which is a power of two limits its efficiency to perform a task of arbitrary size. The *Fibonacci cube* ($FC$) [1], [3] is a special subcube of a hypercube based on the Fibonacci number $F_n = F_{n-1} + F_{n-2}$. It has been shown that the Fibonacci cube can efficiently simulate many hypercube algorithms. The Fibonacci cube uses about 1/5 fewer links than the comparable hypercube and its size does not increase as fast as the hypercube. Wu [9] proposed *Extended Fibonacci Cubes* ($XFC$) by changing the initial values of the Fibonacci recursive formula. $XFC$s are better than $FC$s in emulating other topologies. Both $FC$s and $XFC$s can be viewed as hypercubes with faulty nodes.

The *enhanced Fibonacci cube* proposed in the paper [7] provides more choices of network size to the family of cube-based structures. The enhanced Fibonacci cube ($EFC$) is defined based on the sequence $F_n = 2F_{n-2} + 2F_{n-4}$ and it maintains virtually all the desirable properties of the Fibonacci cube. Moreover, it has several desirable network properties such as Hamiltonian property that the Fibonacci cube does not have. We show that $EFC$ is superior to the Fibonacci cube in following aspects: (1) $EFC$ includes $FC$ of the same order as a subgraph. (2) All $EFC$s are Hamiltonian while more than 2/3 of $FC$s are not. (3) $EFC$ can embed the tree network with dilation 1, while no dilation 1 embedding of trees in $FC$s with comparable expansion is known. Data communication among processors in a multicomputer is very common. In this paper we discuss three basic types of data routing in the enhanced Fibonacci cube, one-to-one (unicast), one-to-all (broadcast) and one-to-many (multicast).

## 2 The Enhanced Fibonacci Cube and Tree

In the following definition, $||$ denotes a concatenation operation, e.g. $10||\{0,1\} = \{100, 101\}$ and $00||\{\} = 00$, where $\{\ \}$ denotes an empty set. Sometimes we may leave out "$||$" for simplicity.

**Definition 1**: Let $EFC_n = (V_n, E_n)$ denote the enhanced Fibonacci cube of the order $n$, then $V_n = 00||V_{n-2} \cup 10||V_{n-2} \cup 0100||V_{n-4} \cup 0101||V_{n-4}$. Two nodes in $EFC_n$ are connected by an edge in $E_n$ if and only if their labels differ in exactly one bit position. As initial values for recursion, $V_3 = \{1, 0\}$, $V_4 = \{01, 00, 10\}$, $V_5 = \{001, 101, 100, 000, 010\}$, and $V_6 = \{0001, 0101, 0100, 0000, 0010, 1010, 1000, 1001\}$.

Note that in $EFC_n$, nodes are ordered in a Gray



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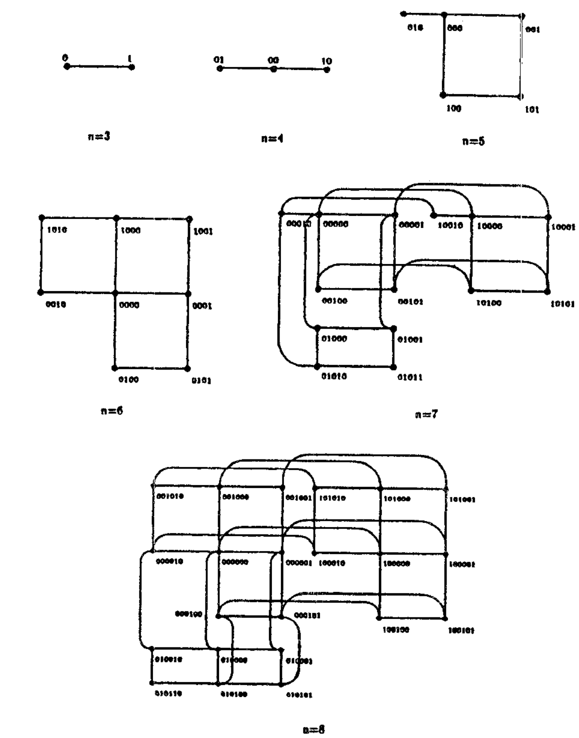

Figure 1: Topological structures of $EFC_n$s, $3 \le n \le 7$

code sequence, and $n-2$ bits are used in a node address. Figure 1 shows $EFC$s with the order of 3, 4, 5, 6, 7 and 8. In the following we list the properties of the enhanced Fibonacci cube, the reader may refer to [7] for details and proofs to these properties.

**Property 1**: *$FC_n$ is a subgraph of $EFC_n$.*

**Property 2**: *For any $n \ge 6$, $EFC_n$ is a Hamiltonian graph.*

**Property 3**: *$EFC_n$ ($n > 6$) can be decomposed into two $EFC_{n-2}$'s and $EFC_{n-4}$'s, the four subgraphs are disjoint.*

**Property 4**: *There exists a Hamming distance path between any two nodes in $EFC_n$.*

**Property 5**: *The diameter of $EFC_n$ is $n-2$.*

**Property 6**: *The node degree of a node in $EFC_n$ is between $\lceil \frac{n}{4} \rceil$ and $n-2$.*

**Definition 2**: An *enhanced Fibonacci tree of order* $n$, denoted by $EFT_n$, is composed of two $EFT_{n-2}$'s and two $EFT_{n-4}$'s, where the root of one $EFT_{n-2}$ is selected as the root of $EFT_n$, the roots of one $EFT_{n-4}$ and the other $EFT_{n-2}$ become the children of the root of the former $EFT_{n-2}$, and the root of the other $EFT_{n-4}$ becomes the child of the root of the former $EFT_{n-4}$. As the basis, $EFT_3$, $EFT_4$, $EFT_5$ and $EFT_6$ are the same as those Fibonacci trees of the same order, respectively.

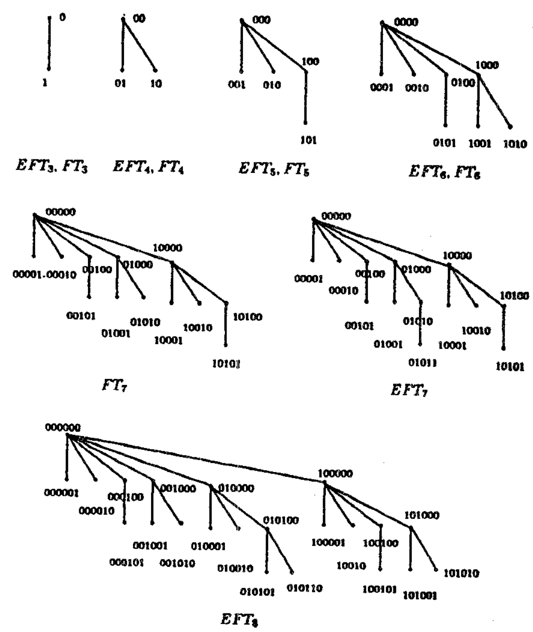

Figure 2: Fibonacci trees ($FT$s) and enhanced Fibonacci trees ($EFT$s).

Figure 2 shows the examples of Fibonacci trees and enhanced Fibonacci trees. A list of the enhanced Fibonacci tree properties are given below. Refer to [8] for more detail and proofs to these claims.

**Property 7**: *$EFT_n$ is a spanning tree of $EFC_n$.*

**Property 8**: *$EFT_n$ can be decomposed into two $EFT_{n-2}$'s and two $EFT_{n-4}$'s, the four subtrees are disjoint.*

**Property 9**: *The span of $EFT_n$ is $n-2$.*

**Property 10**: *The height of $EFT_n$ is $\lceil \frac{n-2}{2} \rceil$.*

**Property 11**: *In $EFT_n$, the children of the root are dimension ordered, i.e. the ith child of the root is the neighbor of the root on the ith dimension.*

**Property 12**: *The pre-order of $EFT_n$ is the same as the order by the binary values of node addresses.*

## 3 Unicast, Multicast, and Broadcast in $EFC_n$

Data communication is the delivery of message from the source to the destination. In general, there are four types of communications based on the sets of the source and destination nodes, *one-to-one*, *one-to-all*, *one-to-many* and *many-to-many*. *Unicast* (one-to-one): The transmission of the same message from a single source node to a single destination node.

*Broadcast* (one-to-all): The transmission of the same message from a single source node to all the other nodes. *Multicast* (one-to-many): The transmission of the same message from a single source node to a subset of destination nodes.

### 3.1 Unicast

According to Property 4, a Hamming distance path exists between any two nodes in $EFC_n$. Hence, it is possible for one-to-one routing to keep the time and traffic steps the same as those for routing on the comparable hypercube $HC_{n-2}$.

The idea used for the enhanced Fibonacci cube is very similar to that of the hypercube. The source or each intermediate node compares its address with that of the destination and obtains the set of the dimensions on which the two addresses differ ($dim\_differ\_set$). In the hypercube it just selects any dimension from the set and sends the routing message to its neighbor along the selected dimension. In the enhanced Fibonacci cube, however, the neighbor along that dimension may not exist (recall that $EFC_n$ is an incomplete hypercube). Although the number of disjoint Hamming distance paths between two nodes is less than that in the comparable hypercube, there still exists a neighbor (along some dimension) that is on a Hamming distance path based on the Property 4. Let $dim\_neighbor\_exist\_set$ denote the set of dimensions along which the neighbors of the node exist. By intersecting the two sets, $dim\_differ\_set$ and $dim\_neighbor\_exist\_set$, a new set $dim\_to\_hop\_set$ is obtained. Property 4 ensures it is not empty. The $dim\_to\_hop\_set$ contains all the dimensions on which the addresses of the current and the destination nodes differ, and along which the neighbors of the current node exist. Then a dimension can be selected from the $dim\_to\_hop\_set$, and routing information is transmitted on from the current node to its neighbor along the dimension.

In the following algorithm, $self\_add$ and $dest\_add$ are addresses of the current node and the destination, respectively.

*Unicast* algorithm /* one-to-one routing in $EFC_n$ */
For the source and each intermediate node do:

- $dim_differ_set \leftarrow \{i \mid i$th bit of $self\_add \oplus dest\_add$ is $1\}$;
- If $dim\_differ\_set = \emptyset$ then the destination is reached else
  - ◇ $dim\_to\_hop\_set = dim\_neighbor\_exist\_set \cap dim\_differ\_set$;
  - ◇ Randomly select a dimension $i \in dim\_to\_hop\_set$;
  - ◇ Send routing message including the destination address to the neighbor along the dimension $i$.

**Theorem 1** *The proposed unicast algorithm on $EFC_n$ is time (traffic) optimal.*

### 3.2 Broadcast

As binomial trees can be used to broadcast on hypercubes, enhanced Fibonacci trees can be used to broadcast on enhanced Fibonacci cubes. However, there is a difference. In a healthy hypercube, each node can be selected as a root of a binomial tree. For the source node of a broadcast, a binomial tree can be constructed under the source node. This is not true in an enhanced Fibonacci cube. In fact, only node $00\cdots0$ can be the root of the enhanced Fibonacci tree, since all other nodes have less than the full number $(n-2)$ of neighbors. If the source is the root node $00\cdots0$, then it can starts broadcast directly as in a hypercube. If the source is a node other than the node $00\cdots0$, it needs several routing steps (up to $\lceil\frac{n-2}{2}\rceil$, the height of the enhanced Fibonacci tree) to first send the broadcast message up to the root, then broadcast the message from the root. To avoid redundant message delivery, the root node does not transmits the message back to the child who sent the message. The source then needs to send the message to its descendants, the nodes in the subtree with the source node as a root. A tag is included in the routing information to indicate the direction (up or down) of message transmission. Following these ideas, we have the broadcast algorithm as shown below.

*Broadcast* algorithm /* one-to-all routing in $EFC_n$ */
For the source node do:

- Send the message with an *up* tag to its parent unless it is a root;
- Send the message with a *down* tag to its children unless it is a leaf;

For each node receiving the broadcast message do:

- If the tag is *up* then
  - ◇ Send the message with an *up* tag to its parent unless it is a root;
  - ◇ Send the message with a *down* tag to its children (except the one who sent the message) unless it is a leaf;
- If the tag is *down* then
  - ◇ Send the message with a *down* tag to its children unless it is a leaf.

Figure 3 shows the stages of a broadcast from the source 01000 in $EFC_7$.

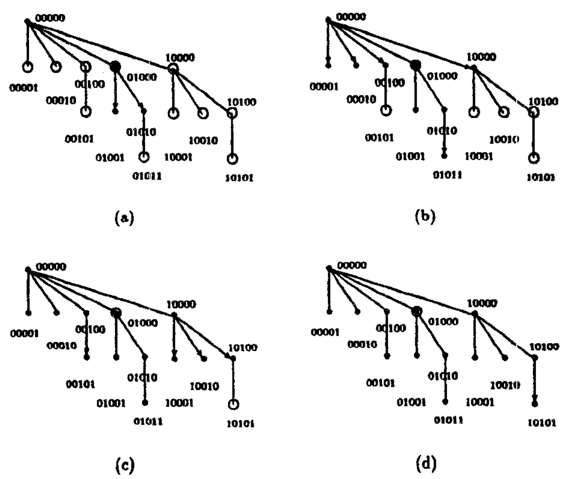


Figure 3: Broadcasting with the source 01000 on $EFC_7$ in four steps.

**Lemma 1** *For $EFC_n$, the above broadcast algorithm has time steps $2\lceil\frac{n-2}{2}\rceil$ and traffic steps $v_n - 1$ ($v_n$ is the number of nodes in $EFC_n$).*

**Theorem 2** *The broadcast algorithm for $EFC_n$ is traffic optimal for any $n$, and is time optimal when $n$ is even and one more step than the optimal time steps when $n$ is odd.*

### 3.3 Multicast

Two multicast schemes are studied. The first one makes use of the enhanced Fibonacci tree. The multicast message is transmitted up and/or down along the branches of the tree as in the broadcasting algorithm except that not all nodes are to be reached. A multicast packet ($msg$, $D$) is used, where $msg$ is the multicast message, and $D$ is the set of destination nodes. When a local node $a$ in the enhanced Fibonacci tree receives a multicast packet, it checks each destination node $d$ in the set $D$. If the node $d$ is $a$, a copy of the message is kept at the node $a$. If the node $d$ is a descendant of $a$, it is added to a destination set that will be forwarded, along with the multicast message, to a child $i$ of $a$, such that the node $d$ is a descendant of this child $i$ or the child $i$. Otherwise, the node $d$ is an ancestor of $a$ or is not directly related to $a$, and it is added to the destination set that will be sent to the parent of $a$.

The second multicast scheme employs the Hamiltonian property of the enhanced Fibonacci cube. The basic idea comes from [6]. In this approach, multicast message is transmitted along a Hamiltonian cycle instead of the enhanced Fibonacci tree. Each node receiving the multicast message chooses next node that is closest in the cycle to the next destination until the last destination receives the message.

## 4 Conclusions

We have discussed unicast, multicast and broadcast in the enhanced Fibonacci cube. The unicast algorithm is time and traffic optimal since it generates a Hamming distance path. The broadcast algorithm uses the enhanced Fibonacci tree to transmit broadcast message. It is traffic optimal for any dimension $n$, and is time optimal when $n$ is even and one step more than optimal time steps when $n$ is odd. We have provided two heuristic multicast algorithms which employ the enhanced Fibonacci tree and a Hamiltonian cycle, respectively.

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