COT 6401 The Analysis of Algorithms

`Final Test`

Open books and notes

Name ——————————————- SSN ——————————————-

1. (**25%, Amortized Analysis**) In table expansion, we expand the table by allocating a new table with more slots than the old table had. In the new heuristic, we allocate a new table that has *four times as many slots as the old one.* Determine the amortized cost of a single insertion using (a) aggregate analysis, (b) the accounting method, and (c) the potential method.

2. (**25%, Selection and Adversary Arguments**) All elements in a given array $A[1..n]$ are identical except one (called distinguished element).

   (a) Design an algorithm that finds the distinguished element using $\lceil \frac{n+1}{2} \rceil$ comparisons.

   (b) Argue that $\lceil \frac{n+1}{2} \rceil$ is the tight bound for this problem. (Your arguement is not restricted to adversary arguments.)

   (c) Determine the average (expectation) number of comparisons of your algorithm. Assume that the location of the distinguished element is a random number within $[1..n]$.

3. (**25%, Probablistic Analysis**) In the hiring problem, we change the notion of "better" as follows: Assign scores $1, 2, .., n$ to n candidates. Each candidate gets a distinct score. Candidate i is better than candidate j if $score(i) > score(j) + 1$. Let $E_{old}[X]$ and $E_{new}[X]$ be the expected numbers of times we hire a new office assistant in HIRE-ASSISTANT(n) (page 92) and HIRE-ASSISTANT(n) under the new notion of "better", respectively.

   (a) Determine $E_{new}[X] - E_{old}[X]$. (Hint: Calculate $Pr\{\text{candidate } i \text{ is hired}\}$ and pay attention to the distribution of the candidate with score $score(i) - 1$.)

   (b) Verify your conclusion by determining $E_{new}[X]$ and $E_{old}[X]$ for $n = 3$ through enumerating all cases.

4. (**25%, Approximation Algorithms**) Given a $2 \times n$ rectangle of 1×1 squares. Some squares have black bullets and others are blank. Use a minimum number of 1×2 rectangles to *cover* all 1×1 squares with black bullets. An approximation algorithm is designed as follows: Randomly select two adjacent black bullets and cover them with a 1×2 rectangle. The process is repeated until only isolated bullets are left. Then each bullet is covered by a 1×2 rectangle.

   (a) Proof that it is a 1.5-approximation algorithm.

   (b) Show a case where the algorithm generates the exact 1.5 approximation.

   (c) If the rectangle is changed from $2 \times n$ to $n \times n$, will the result of the approximation algorithm (1.5 approximation) still hold?

5 Extra Points

The notion of "better" is defined the same way as in Problem 3 and is now applied to the on-line hiring problem; that is, the condition on line 6 of ON-LINE-MAXIMUM(k, n) (page 115) is now changed to "$score(i) > bestscore+1$". Provide intuitive explanations to the following questions without formal derivation:

- Let $k = n/e + \Delta$ be the value that maximizes the lower bound on $Pr(S)$. Should Δ be positive or negative? and why?
- Select one of the following: (a) $|\Delta| = 1$, (b) $|\Delta| < 1$, or (c) $|\Delta| > 1$. Justify your selection.