COT 6401 The Analysis of Algorithms

`Midterm Test`

Open books and notes

Name ——————————————- SSN ——————————————-

1. (25%) A k-**combination** of an n-set S is simply a k-subset of S. For example, $\{ab, ac, ad, bc, bd, cd\}$ is the 2-combination of the 4-set $\{a, b, c, d\}$. Write a pseudocode to generate the 3-**combination** of an n-set S. Show the correctness of your code by showing the 3-combination of the 5-set $\{a, b, c, d, e\}$. What is the complexity of your solution?

2. (25%) Modify the HEAPIFY (A, i) (page 143) for a **ternary heap** which is a complete ternary tree where three children of a node are called LEFT, CENTER, and RIGHT. Assume that the heap property remains the same.

3. (25%) Give an efficient algorithm to count the **total number of paths** in a directed acyclic graph (DAG) from u to v. Apply your algorithm to Figure 25.8 (page 537) where $u = r$ and $v = x$.

4. (25%) Given a **green onion** of n inches, you are required to cut it into n pieces of one inch each. Assume that after the onion has been cut into k pieces, they can be "piled" together so that the following cut can generate up to $2k$ pieces. What is the minimum number of **cuts**? Determine a **greedy approach** that generates a minimum number of cuts. Show the correctness of your approach for $n = 13$.

   (**Bonus Points**, 5 pts)

5. In the activity-selection problem of n activities (n is a large integer, say $n > 1,000$), suppose a greedy approach always selects a compatible activity with an **earliest starting time**. Provide a **sufficient condition** such that this greedy approach always generate an optimal solution.