Consider a renewal process. Let $X$ be the interrenewal times; and let $I$ and $R$ be the length of an interval interrupted at random and its remainder, respectively. The following BASIC simulation calculates the average values of $X, I$, and $R$ (based on 10,000 replications of $I$ and $R$, where $T$ is a random interruption point). (A concrete example: X could be the time between successive taxis, T the time of arrival of a customer who wants a taxi, I the time between the taxi that arrived just before T and the taxi that arrives just after T , and R the time that the customer waited for the next taxi.)

```
RANDOMIZE
100 FOR \(\mathrm{j}=1\) TO 10000
\(110 \mathrm{~S}=0\)
\(120 \mathrm{~T}=-1000^{*} \mathrm{LOG}(1-\mathrm{RND})\)
130 [generate X]
\(140 \mathrm{c}=\mathrm{c}+1\)
\(150 \mathrm{SX}=\mathrm{SX}+\mathrm{X}\)
\(160 \mathrm{~S}=\mathrm{S}+\mathrm{X}\)
170 IF \(\mathrm{S}<\mathrm{T}\) THEN 130
180 R=S-T: I=X
190 SR=SR+R: SI=SI+I
200 NEXT j
210 PRINT SX/c, SI/10000, SR/10000
```

Consider the three cases:
(1) $X=2$ (constant)
(2) $\mathrm{P}(\mathrm{X}=1)=0.9$ and $\mathrm{P}(\mathrm{X}=11)=0.1$
(3) $\mathrm{P}(\mathrm{X}=2)=0.9$ and $\mathrm{P}(\mathrm{X}=12)=0.1$

Run the simulation for each case, and fill in the following table.

| $\mathrm{E}(\mathrm{X})$ |  |  |  | $\mathrm{E}(\mathrm{I})$ |  | E (R) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | Theory | Simulation | Theory | Simulation | Theory | Simulation |
| 1 | 2 |  | 2 |  | 1 |  |
| 2 | 2 |  |  |  |  |  |
| 3 | 3 |  |  |  |  |  |

Describe your observations. Show (or explain) how you can calculate the theory values given in the table. Try to explain why the values for $\mathrm{E}(\mathrm{I})$ and $\mathrm{E}(\mathrm{R})$ are so weird (if they are) in cases 2 and 3.

