The prerequisite for this course is MAC 2254 (Calculus for Engineers 2) or MAC 2312 (Calculus with Analytic Geometry II). The purpose of this assignment is to review some topics from these courses that will be used in STA 4821.

A *random variable X* can be described by its *density function* $f_x(t)$ or its *distribution function* $F_x(t)$. They are related as follows.

$$f_x(t) = \frac{d}{dt} F_x(t)$$

$$F_x(t) = \int_{-\infty}^{t} f_x(u)du$$

The first two *moments* of a random variable *X* are

$$E(X) = \int_{-\infty}^{\infty} t \cdot f_x(t)dt$$

$$E(X^2) = \int_{-\infty}^{\infty} t^2 \cdot f_x(t)dt$$

The *convolution* $g(t)$ is defined as

$$g(t) = \int_{-\infty}^{\infty} f_x(u) \cdot f_x(t-u)du$$

Later, these formulas will be given physical interpretations; but for now, take the viewpoint that the following exercise will get the mathematical details out of the way so that later we can concentrate on their meaning rather than their calculation.

Fill in the following table. Show all work on separate pages. Draw the graphs of $f_x(t)$, $F_x(t)$ and $g(t)$ on the graph paper provided.

Take $a$ = 0.7 and $b$ = 1.5. In the second case, write the answers in terms of the parameter $\lambda$, but use $\lambda = 2$ when drawing the graphs and evaluating $\int_a^b f_x(t)dt$ and $\int_a^b g(t)dt$.

| | $E(X)$ | $E(X^2)$ | $F_x(t)$ | $\int_a^b f_x(t)dt$ | $g(t)$ | $\int_a^b g(t)dt$ |
|---|---|---|---|---|---|---|
| $f_x(t) = \begin{cases} 0 & (t<0) \\ 1 & (0 \le t \le 1) \\ 0 & (t>1) \end{cases}$ | | | | | | |
| $f_x(t) = \begin{cases} 0 & (t<0) \\ \lambda e^{-\lambda t} & (t \ge 0) \end{cases}$ | | | | | | |