The following BASIC code simulates the single-server queue with FIFO service. It generates the interarrival times and the service times for 100,000 customers; and it produces estimates of the server utilization, the fraction of customers who must wait in the queue, and the average waiting time.

```
100 FOR i = 1 TO 100000
110 IA =                        (generate interarrival time)
120 T = T + IA
130 W = W + X – IA
140 IF W < 0  THEN W = 0
150 IF W > 0  THEN c = c + 1
160 SW = SW + W
170 X =                         (generate service time)
180 SX = SX + X
190 NEXT i
200 PRINT SX/T, c/100000, SW/100000
```

Adapt the program and run it (using the language of your choice) for four different service-time distributions:

(1) exponential service times, with mean service time $E(X) = 0.5$
(2) constant service time, $X = 0.5$
(3) $X \sim U(0,1)$
(4) $P(X=1/3) = 0.9$, $P(X=2) = 0.1$

Assume that the interarrival times are exponentially distributed with mean value 0.625 (that is, Poisson arrivals with rate $\lambda = 1.6$). Fill in the table. For case (1), draw the graph of the theoretical distribution function of waiting times, $F_w(t)$; and, on the same axes plot the simulation estimates at the values of t as t goes from –1 to 12 in increments of 1, and fill in the corresponding table.

<u>Theory for the M/G/1 queue</u>: If λ is the arrival rate and X is the service time, then the *server utilization* is given by

$$\rho = \begin{cases} \lambda E(X) & \text{if } \lambda E(X) < 1 \\ 1 & \text{if } \lambda E(X) \geq 1 \end{cases}$$

The probability that a customer must wait in the queue is

$$P(W > 0) = \rho,$$

and the mean waiting time is given by the famous *Pollaczek-Khintchine formula*,

$$E(W) = \frac{\rho}{1-\rho} \cdot \frac{E(X)}{2} \cdot \left(1 + \frac{V(X)}{E^2(X)}\right).$$

In addition, if the service times are exponentially distributed, and the service is FIFO, then

$$F_w(t) = \begin{cases} 0 & (t < 0) \\ 1 - \rho e^{-(1-\rho)\cdot\frac{t}{E(X)}} & (t \geq 0) \end{cases}$$

There is no simple formula for $F_w(t)$ when the service times are not exponentially distributed. (Therefore, simulation is an important tool for the analysis of queues whose service times have any arbitrary specified distribution of service times; and the theoretical results for the special case of exponential service times are important because they can be used to check the logic and accuracy of the simulation.)

	ρ		$P(W > 0)$		$P(W > 0.5)$		$E(W)$	
X	theory	simulation	theory	simulation	theory	simulation	theory	simulation
1								
2					NA			
3					NA			
4					NA			

	$F_w(t)$	
t	theory	simulation
-1		
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		