

I pledge my honor that I have neither given nor received aid on this exam.

NAME \_\_\_\_\_

Z NUMBER \_\_\_\_\_

Show all work. Attach work pages. Write on one side of page only. Write answers in space provided. Staple in upper left-hand corner.

Consider two identical-looking dice. One of the dice is fair (six equally-likely faces, numbered 1, 2, ..., 6), but the other die is "loaded" (the face that ordinarily has a 1 has a 6 instead; that is, the loaded die has two 6's, on opposite faces). One of the dice is chosen at random and is rolled three times.

1. Find the probability that the value of the first roll is 6.
2. Find the probability that the value of the second roll is 6.
3. Find the probability that the first roll is 6 and the second roll is 6.
4. Find the probability that the sum of the first two rolls is 8.
5. Find the probability that the die being rolled is the fair die if the sum of the first two rolls is 8.
6. Find the probability that the third roll will be 6 if the sum of the first two rolls was 8.

Write your answers here:

1.            $\frac{1}{4}$           

2.            $\frac{1}{4}$           

3.            $\frac{5}{72}$           

4.            $\frac{1}{6}$           

5.            $\frac{5}{12}$           

6.            $\frac{19}{72}$           

$$P_1 = P(X_1=6) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{6}\right) = \frac{1}{4}$$

$$P_2 = P(X_2=6) = P(X_1=6) = P_1 = \frac{1}{4}$$

$$P_3 = P(X_1=6, X_2=6) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{2}{6}\right)^2 = \frac{5}{72}$$

$$P_4 = P(X_1+X_2=8) = \left(\frac{1}{2}\right)\left(\frac{5}{36}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{36}\right) = \frac{1}{6}$$

$$P_5 = P(X = \text{fair} | X_1+X_2=8)$$

$$= \frac{P(X = \text{fair})P(X_1+X_2=8 | X = \text{fair})}{P(X_1+X_2=8)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{5}{36}\right)}{P_4} = \frac{5}{12}$$

$$P_6 = P_5 \left(\frac{1}{6}\right) + (1-P_5)\left(\frac{2}{6}\right) = \left(\frac{5}{12}\right)\left(\frac{1}{6}\right) + \left(\frac{7}{12}\right)\left(\frac{2}{6}\right) = \frac{19}{72}$$