A Decomposition Theorem for Polling Models: The Switchover Times are Effectively Additive

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A DECOMPOSITION THEOREM FOR POLLING MODELS: THE SWITCHOVER TIMES ARE EFFECTIVELY ADDITIVE

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We consider the classical polling model: queues served in cyclic order with either exhaustive or gated service, each with its own distinct Poisson arrival stream, service-time distribution, and switchover-time (the server's travel time from that queue to the next) distribution. Traditionally, models with zero switchover times (the server travels at infinite speed) and nonzero switchover times have been considered separately because of technical difficulties reflecting the fact that in the latter case the mean cycle time approaches zero as the travel speed approaches infinity. We argue that the zero-switchover-times model is the more fundamental model: the mean waiting times in the nonzero-switchover-times model *decompose* (reminiscent of vacation models) into a sum of two terms, one being a simple function of the sum of the mean switchover times, and the other the mean waiting time in a "corresponding" model obtained from the original by setting the switchover times to zero and modifying the service-time variances. This generalizes a recent result of S. W. Fuhrmann for the case of constant switchover times, where no variance modification is necessary. The effect of these studies is to reduce computation and to improve theoretical understanding of polling models.

A *polling model* is used to represent a system of multiple queues that are attended by a single server that travels from queue to queue in some prescribed manner. These models have many important applications (computer networks, telephone switching systems, materials handling, etc.), and in general, they are extremely complicated. Consequently, there has developed a huge literature dealing with various versions of these models and their numerical analysis. In this paper we consider two of the most basic polling models: queues served in cyclic order, with *exhaustive service* or *gated service* in which the *switchover times* are random variables with arbitrarily prescribed distributions. We show that the mean waiting times in these models enjoy a *decomposition* into a sum of two terms: (1) the mean waiting time in a "corresponding" model in which the switchover times are zero, and (2) a simple term that relates only to the mean switchover times. This decomposition, which is reminiscent of a similar result for vacation models (and, indeed, is closely related to it), greatly reduces computational difficulties and gives theoretical insight that should prove useful in the analysis (mathematical and numerical) of other polling models.

In Section 1 we define the model and state our decomposition theorem; in Section 2 we describe the background and motivation behind the theorem; and in Section 3 we give the proof.

1. THE MODEL AND THE DECOMPOSITION THEOREM

A single server serves in cyclic order a sequence of N infinite-capacity queues. Queue i ($i = 1, 2, \ldots, N$) receives Poisson arrivals at rate λ_i; has a service-time distribution with mean service time b_i and second moment $b_i^{(2)}$; and has a switchover-time (the time required for the server to travel from queue i to queue $i + 1$) distribution with mean r_i and variance δ_i^2. The arrival times, service times, and switchover times are all mutually independent, and the queue discipline is nonbiased (i.e., at each queue, the next customer selected for service does not depend on that customer's service time).

We consider two different polling disciplines: exhaustive service and gated service. With exhaustive service, the server switches from queue i to queue $i + 1$ only when there are no customers remaining in queue i; and with gated service, the server closes a "gate" behind the waiting customers when it arrives at queue i and switches to queue $i + 1$ upon completion of service of all the customers in front of the gate.

Let $\rho_i \equiv \lambda_i b_i$, $\rho \equiv \rho_1 + \ldots + \rho_N$, and $R \equiv r_1 + \ldots + r_N$. Let W_i be the waiting time (from arrival epoch to start of service) of a customer in queue i, and let W_i^0 be the corresponding variable in the "corresponding" 0-switchover-times model; this is defined to be the model that differs

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from the original only in that (1) $\mathbf{r} \equiv (r_1, \ldots, r_N) = \mathbf{0}$ (and thus $\boldsymbol{\delta}^2 \equiv (\delta_1^2, \ldots, \delta_N^2) = \mathbf{0}$), where $\mathbf{0}$ denotes a vector of zeros, and (2) $\mathbf{b}^{(2)} \equiv (b_1^{(2)}, \ldots, b_N^{(2)})$ is replaced by a vector of *correspondence parameters* $\mathbf{x}^{(2)} \equiv (x_1^{(2)}, \ldots, x_N^{(2)})$, as defined in (2) or (4) below. (This paper restricts its attention to the expected values of the waiting times; consequently, the service-time and switchover-time moments higher than second order are irrelevant.) Finally, assume that $\rho < 1$ (which guarantees stability) and that the system is in statistical equilibrium. Then we have the following theorem.

Theorem. *(1) If the polling discipline is exhaustive service, then*

$$E[W_i] = E[W_i^0] + \frac{R}{2}\frac{1-\rho_i}{1-\rho}, \tag{1}$$

where W_i^0 is the waiting time in the "corresponding" exhaustive-service model with zero switchover times and correspondence parameters

$$x_i^{(2)} = b_i^{(2)} + \delta_{i-1}^2\left(\frac{\lambda_i R}{1-\rho}\right)^{-1}. \tag{2}$$

(2) If the polling discipline is gated service, then

$$E[W_i] = E[W_i^0] + \frac{R}{2}\frac{1+\rho_i}{1-\rho}, \tag{3}$$

where W_i^0 is the waiting time in the "corresponding" gated-service model with zero switchover times and correspondence parameters

$$x_i^{(2)} = b_i^{(2)} + \delta_i^2\left(\frac{\lambda_i R}{1-\rho}\right)^{-1}. \tag{4}$$

The theorem says that in each case, the expected waiting time in the general-switchover-times model equals the sum of the expected waiting time in the "corresponding" 0-switchover-times model plus a simple term that depends only on the server utilization and the sum (but not the individual values) of the mean switchover times.

In the important special case when the switchover times are (not necessarily equal) constants (i.e., when $\boldsymbol{\delta}^2 = \mathbf{0}$), then, from (2) and (4), $x_i^{(2)} = b_i^{(2)}$ and the "corresponding" 0-switchover-times model is the "true" corresponding 0-switchover-times model; that is, the effect of the switchover times is truly additive. During the course of our work, we learned via a preprint of Fuhrmann (1992) that this special case of the theorem was about to be published (and, in fact, had been discovered in 1986). Fuhrmann's argument uses the concept of "ancestor" and "ancestral lines" in the same way as used in Fuhrmann and Cooper (1985) in the analysis of decomposition in the $M/G/1$ generalized-vacations model. Fuhrmann shows that, for constant switchover times, the population of customers present in the system (represented by a vector whose components are the numbers of customers present at each queue) at a polling epoch enjoys a stochastic decomposition. In contrast, our proof (for the exhaustive-service case) begins with the vacation decomposition result itself, in its mean-value form. Our argument, which is completely different from and independent of Fuhrmann's, not only provides some additional insight for the analysis of other polling models, but it also leads to a surprising generalization (for mean values) relative to Fuhrmann's statement. That is, the decomposition theorem retains its form *even when the switchover times are random variables.*

When the switchover-time variances are nonzero, (2) suggests that the variance of the switchover time from queue $i - 1$ to i can be effectively "absorbed" into the service-time variance at queue i, as follows: It is well known that when $R > 0$, the expected number of customers served at queue i per cycle is equal to $\lambda_i R/(1 - \rho)$ (see, e.g., Kuehn 1979). Thus, the second term in (2) can be interpreted as the fraction of the switchover-time variance apportioned to each customer served during that visit, and hence the "new" service-time variance is the sum of the original service-time variance and the apportioned switchover-time variance. A similar interpretation applies to (4). This notion of variance absorption is, in fact, closely related to observations made previously by Ferguson and Aminetzah (1985) and Sarkar and Zangwill (1991).

2. BACKGROUND AND MOTIVATION

Cooper and Murray (1969) studied a polling model of $N \geq 2$ $M/G/1$ queues served in cyclic order. The application that motivated that paper was the electronic telephone-switching system; there the switchover times are negligible in comparison with the service times, so it was natural to direct attention to the 0-switchover-times model. This work was continued in Cooper (1970), where a general "vacation" model was defined; the key idea was that, in the polling model, from the viewpoint of a particular queue, the time spent serving the other queues can be interpreted as a vacation. Therefore, the FIFO waiting times "decompose" into a sum of two terms, one of which corresponds to the same model but without vacations (i.e., the ordinary $M/G/1$ queue), with the other term relating only to the lengths of the vacations. (See, e.g., Fuhrmann and Cooper, Doshi 1990, and Takagi 1991b.) The existence of decomposition often simplifies the analysis of vacation models in general, and polling models in particular. (See, e.g., Takagi 1986 and 1990, whose notation we adopt.) In the present paper, we show that in certain polling models with general switchover times, an analogous decomposition (relative to switchover times) occurs, even though the well-known sufficient conditions for decomposition (as stated, e.g., in Fuhrmann and Cooper) are not met.

The first natural generalization of the 0-switchover-times model is to let the switchover times be nonnegative random variables; landmark papers in this direction include Hashida and Nakamura (1969), Eisenberg (1972), Hashida (1972), Aminetzah (1975), Humblet (1978), Ferguson and Aminetzah (1985), and Sarkar and Zangwill (1989), and also (in discrete time) Konheim and Meister (1974) and Swartz (1980). With the availability of results for the general-switchover-times models, the 0-switchover-times models seemed less important, for two reasons: (1) in principle, the 0-switchover-times model is subsumed as a special case of the general-switchover-times model, and (2) many of the motivating applications, such as computer networks (see, e.g., Takagi 1991a) and transportation and traffic-control systems (see, e.g., Daganzo 1990) have switchover times that cannot be neglected. Consequently, only a few papers have been published that restrict their attention to 0-switchover-times models (we mention Blanc 1990, Fuhrmann and Moon 1990, and Takagi 1987, 1989).

Interestingly, the analysis of Cooper and Murray of the 0-switchover-times models is conceptually more difficult than that of their general-switchover-times counterparts. This is because in a "natural" formulation of the 0-switchover-times model, the server will execute an infinite number of cycles during any (finite) period throughout which the system is empty; as a consequence, the mean cycle time is zero. This difficulty was first observed by Eisenberg (1972) (who has recently returned to this question; see Eisenberg 1994). Takagi (1990) observes that when explicit formulas exist for the general case, the corresponding results for the 0-switchover-times case can be obtained by taking the limit $R \to 0$ (and $(\delta_1^2 + \ldots + \delta_N^2)/R \to 0$), but, "Otherwise we need separate analysis for systems with zero switchover times." (He provides this analysis in Chapter 7 of Takagi 1986, and in more complete form in Takagi 1987, which includes a more elegant and economical version of Cooper and Murray, and Cooper. Significantly, Takagi 1987 is not officially published, although a shorter version, Takagi 1989, is.) Finally, this question is at the heart of a recent paper by Levy and Kleinrock (1991). The results of our paper support the centrality (and ultimate simplicity) of the 0-switchover-times model.

It would be interesting, and perhaps useful, to see whether other polling models allow a decomposition of the form $E[W_i] = E[W_i^0] + f_i(R)$. For example, consider the polling model in which the queues are classified into two sets: those belonging to set E are served exhaustively, while those belonging to set G are gated (see Takagi 1989). Then, the results of Srinivasan (1991) imply that, for constant switchover times, the mean waiting time for queue i is given by (1) if queue i is in E, and by (3) if queue i is in G, where $E[W_i^0]$ describes the corresponding mixed polling model with zero switchover times. We intend also to investigate other variations and extensions (e.g., the models considered in Baker and Rubin 1987, Choudhury 1990, Eisenberg 1972, Konheim et al. 1994, Levy and Sidi 1991, and Takagi 1990) in subsequent work.

3. PROOF

In the interest of brevity, we will only outline our arguments here (for details, see Cooper et al. 1993, which we refer to hereafter as CNS). We first give the proof for the exhaustive-service case, which begins with an expected-value version of the decomposition theorem for $M/G/1$ queues with generalized vacations (Fuhrmann and Cooper, Proposition 3 and Equation (5), p. 1125) for exhaustive service:

$$E[W_i] = \frac{\lambda_i b_i^{(2)}}{2(1-\rho_i)} + E[\tilde{V}_i], \tag{5}$$

where the first term on the right-hand side of (5) is the Pollaczek-Khintchine formula for the mean waiting time in an ordinary $M/G/1$ queue, and $\tilde{V}_i$ is the forward-recurrence time of a "vacation" V_i from queue i. If we define an intervisit time I_i as the time interval from an instant at which the server begins its switchover time from queue i until the instant at which it next completes its switchover time from queue $i-1$, then $V_i \equiv I_i$ and

$$E[\tilde{V}_i] = \frac{E[I_i^2]}{2E[I_i]} = \frac{1}{2}E[I_i] + \frac{V[I_i]}{2E[I_i]}, \tag{6}$$

where $V[\cdot]$ denotes a variance. Substituting (6) into (5) gives

$$E[W_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \frac{\lambda_i b_i^{(2)}}{2(1-\rho_i)} + \frac{1}{2}E[I_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] + \frac{V[I_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}{2E[I_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}, \tag{7}$$

where, for clarity, we have explicitly denoted the dependencies on $\mathbf{b}^{(2)}$, R, and $\boldsymbol{\delta}^2$.

We now temporarily restrict our attention to the case of constant switchover times, i.e., $\boldsymbol{\delta}^2 = \mathbf{0}$. In CNS, we derive (beginning with Equations (3.20) and (3.21a, b, c) in Takagi 1990, which are due originally to Ferguson and Aminetzah, Aminetzah, and Humblet) the following interesting result: For any $R > 0$,

$$\frac{V[I_i(\mathbf{b}^{(2)}, R, \mathbf{0})]}{2E[I_i(\mathbf{b}^{(2)}, R, \mathbf{0})]} = c_i(\mathbf{b}^{(2)}); \tag{8}$$

that is, with constant switchover times, the intervisit-time variance-to-mean ratio is, for every i, independent of the individual switchover times.

It is well known (see, e.g., Takagi 1990, Equation (3.3)) that for $R > 0$,

$$E[I_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \frac{1-\rho_i}{1-\rho}R. \tag{9}$$

Substitution of (9) and (8) into (7) yields

$$E[W_i(\mathbf{b}^{(2)}, R, \mathbf{0})] = \frac{\lambda_i b_i^{(2)}}{2(1-\rho_i)} + \frac{R}{2}\frac{1-\rho_i}{1-\rho} + c_i(\mathbf{b}^{(2)}). \tag{10}$$

Moreover, by letting $R \to 0$ in (10) (see Takagi 1990, p. 294), we have

$$E[W_i(\mathbf{b}^{(2)}, 0, \mathbf{0})] = \frac{\lambda_i b_i^{(2)}}{2(1-\rho_i)} + c_i(\mathbf{b}^{(2)}). \tag{11}$$

Comparison of (10) and (11) shows that

$$E[W_i(\mathbf{b}^{(2)}, R, \mathbf{0})] = E[W_i(\mathbf{b}^{(2)}, 0, \mathbf{0})] + \frac{R}{2}\frac{1-\rho_i}{1-\rho}; \tag{12}$$

and this proves (1) for the case of constant switchover times.

To complete the proof of the theorem for exhaustive service, we now drop the assumption that $\boldsymbol{\delta}^2 = \mathbf{0}$. In CNS, we show (using (7) and Equations (28) and (29) in Ferguson and Aminetzah) that, with $\mathbf{x}^{(2)}$ defined according to (2),

$$E[W_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = E[W_i(\mathbf{x}^{(2)}, R, \mathbf{0})]; \tag{13}$$

and this, together with (12) (with $\mathbf{x}^{(2)}$ replacing $\mathbf{b}^{(2)}$), proves (1).

Also, as observed by a referee, the first part of the theorem can be proved using results in Sarkar and Zangwill (1989, 1991). Define a cycle time C_i as the time between two successive instants at which the server switches from, or *leaves*, queue i. Then (Takagi 1986),

$$E[W_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \frac{1-\rho_i}{2}\left[E[C_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] + \frac{V[C_i^2(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}{E[C_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}\right], \tag{14}$$

where (Kuehn)

$$E[C_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \frac{R}{1-\rho}. \tag{15}$$

Sarkar and Zangwill (1991) state (Equation (2.2.7))

$$V[C_i^2(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \sum_{j=1}^{N} h_{ij}(\rho_1, \ldots, \rho_N)\left[\lambda_j b_j^{(2)}\frac{R}{1-\rho} + \delta_{j-1}^2\right], \tag{16}$$

where the functions h_{ij} can be obtained from Equations (4.6)–(4.12) in Sarkar and Zangwill (1989), and can be shown to depend only on the server utilizations. Substituting (15) and (16) into (14) gives

$$E[W_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = \frac{1-\rho_i}{2}\left\{\frac{R}{1-\rho} + \sum_{j=1}^{N} h_{ij}(\rho_1, \ldots, \rho_N) \cdot \left[\lambda_j b_j^{(2)} + \delta_{j-1}^2\left(\frac{R}{1-\rho}\right)^{-1}\right]\right\}. \tag{17}$$

It follows easily from (17) that both (12) and (13) hold; and this again proves (1). (Also, Sarkar and Zangwill 1991, p. 450, Section 4.1, discuss "variance interchange," which is related to our (13).)

The argument for the second part of the theorem, for gated service, is similar. Now define a cycle time C_i as the time between two successive instants at which the server polls, or *arrives at*, queue i. Takagi (1990) gives (Equation (3.27b), which is due originally to Aminetzah)

$$E[W_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)] = (1+\rho_i)\frac{E[C_i^2(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}{2E[C_i(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]}. \tag{18}$$

Using (18) and formulas for $E[C_i^2(\mathbf{b}^{(2)}, R, \boldsymbol{\delta}^2)]$ given in Appendix B in Ferguson and Aminetzah (with corrections in Choudhury and Takagi 1990), we show in CNS that

$$E[W_i(\mathbf{b}^{(2)}, R, \mathbf{0})] = E[W_i(\mathbf{b}^{(2)}, 0, \mathbf{0})] + \frac{R}{2}\frac{1+\rho_i}{1-\rho}; \tag{19}$$

and moreover, with $\mathbf{x}^{(2)}$ defined by (4), (13) holds for gated service also (again, these could be proved using results in Sarkar and Zangwill 1989). Comparison of (13) and (19) (with $\mathbf{x}^{(2)}$ replacing $\mathbf{b}^{(2)}$) proves (3).

Note added in proof: In a subsequent paper (Srinivasan et al. 1995), we have, among other things, extended the results of the present paper to describe the relationship between the waiting-time distributions (as opposed to only the expected values) in the zero- and the nonzero-switchover-times models; this relationship can be used to give a different proof of the present paper's theorem.

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