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TITLE— Analysis of Alternate Routing Networks with Account Taken of the Nonrandomness of Overflow Traffic

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ABSTRACT

An iteration procedure that takes into account the nonrandomness of overflow traffic has been developed to compute the link blocking probabilities for alternate routing networks. The procedure is contrasted with a similar method in which the nonrandomness of overflow traffic is neglected. Both methods are applied to a 7-node hierarchical network under several different traffic conditions and the results are compared with those obtained by simulation.

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SUBJECT: Analysis of Alternate Routing Networks with Account Taken of the Nonrandomness of Overflow Traffic

DATE: NOV 25 1964

FROM: R. B. Cooper
S. S. Katz

MM-64-3122-2

MEMORANDUM FOR FILE

Introduction

Because of the complexity of the problem, the analysis of communications networks that permit alternate routing of calls has been performed largely by simulation techniques. Although flexible and accurate, simulations require a great deal of computer time and are therefore somewhat limited in their scope of application. Moreover, they are generally unable to provide as much insight into the causal relationships present as do analytic solutions. An analytic method in which the nonrandomness of overflow traffic is considered has been devised to evaluate the traffic performance of alternate routing networks.

The proposed method is an iteration procedure in which the traffic offered to the i^{th} trunk group is characterized by the mean a_i and variance v_i of the distribution of simultaneous calls. Traffic which cannot be carried on the i^{th} trunk group has an overflow distribution with mean and variance denoted by α_i and β_i, respectively. Associated with each trunk group is a probability of blocking, B_i, which is defined as the ratio of α_i to a_i. During an iteration, new values are computed for a_i, v_i, α_i, β_i and B_i. As these characteristic values for the i^{th} trunk group change, other trunk groups are affected, resulting in corresponding changes in their characteristic values. The process is continued until the change in each link blocking probability B_i from one iteration to the next is less than a preassigned value.

When the iteration procedure has converged, the link blocking probabilities may then be suitably combined to obtain an estimate of the point-to-point loss probability between each node pair.

The analytic method is demonstrated in this memorandum for networks employing a hierarchical routing structure. However, the method can be applied to a variety of other routing plans, such as symmetrical and ring routing.

Assumptions

The problem is studied under the following assumptions:

(1) Traffic originates according to a Poisson process.

(2) The system is in statistical equilibrium.

(3) Overflow traffic is adequately characterized by the mean and variance of its distribution.

(4) The occupancy distributions of the trunk groups are statistically independent of each other.

(5) Any call attempt that is unable to find an idle path to its destination is cleared from the system and does not return.

The first two assumptions, those of random (Poisson) arrivals and statistical equilibrium, are commonly made in traffic studies and will not be discussed here.

The third assumption is the basis for the use of the "equivalent random" (ER) method[1] for describing overflow traffic. The method has been widely used and is considered quite accurate.

The fourth assumption is used in the following two ways:

(i) The probability that two or more given trunk groups are all simultaneously blocked is assumed equal to the product of their individual blocking probabilities.

(ii) The variance of the distribution of a parcel of overflow traffic composed of two separate components of overflow with variances v_1 and v_2 is assumed to be the sum v_1+v_2; that is, any covariance is neglected.

Assumption (5) is commonly employed in traffic studies. Retrials may be accounted for by assuming that they occur in such a way that the offered traffic is still described by a Poisson distribution. The offered loads may then be adjusted to reflect the inclusion of retrials.

Description of Method of Solution

The method of analysis consists of an iteration procedure in which, at each step, the overflow traffic from each link is computed. The overflow parcels from all the links are then suitably combined in accordance with the particular alternate routing procedure of the network. The load offered to each particular link, consisting of a direct load (random) and a combination of overflow parcels (nonrandom), is converted into an "equivalent random" (ER) load, allowing the overflow to again be computed. The process is repeated until the changes in the computed quantities from the previous iteration step are less than some preassigned amount.

Used in the analysis are two techniques that will now be described:

(1) the "equivalent random" (ER) procedure for handling nonrandom traffic, and

(2) a method of apportioning the variance of an overflow distribution.

The ER Method[1]

It is well known that when random (Poisson) traffic is offered to a loss [redacted] system, the overflow is nonrandom; that is, it cannot be accurately described by a Poisson distribution. In order to determine the loss when nonrandom traffic is offered to a full access group, the ER method is used.

Suppose that a random load (with mean) A is submitted to S full access trunks, and suppose that every call that finds all trunks busy overflows to an infinite trunk group. The statistical equilibrium probability that an arbitrary call is blocked on the first S trunks is given by the well known Erlang B formula,

$$B(S,A) = \frac{A^S/S!}{\sum_{n=0}^{S} A^n/n!} \qquad (1)$$

Let N be the number of occupied trunks in the overflow group with expectation E(N) = a and variance V(N) = v. For any random load with mean A and any group of S trunks, there corresponds an overflow distribution with mean a and variance v, where

$$a = AB(S,A) \tag{2}$$

$$v = a\left[1-a+\frac{A}{S+1+a-A}\right] \tag{3}$$

In other words, to every pair (A,S) there corresponds the (unique) pair (a,v). There are, of course, higher moments, but it is assumed in the ER method that the overflow distribution is adequately described by the two parameters, (2) and (3).

Conversely, any nonrandom load (a,v) can be considered as the unique overflow from the random load A and S trunks, (A,S), where S will not generally be an integer. The overflow that occurs when a group of c trunks is offered the nonrandom load (a,v) is taken to be the same as the overflow that occurs when the random load A is offered to a group of (S+c) trunks. Therefore, the mean α and variance β of the load overflowing the c trunks are given by equations (2) and (3),

$$\alpha = AB(c+S,A) \tag{4}$$

$$\beta = \alpha\left[1-\alpha+\frac{A}{c+S+1+\alpha-A}\right] \tag{5}$$

The probability of blocking on the original group of c trunks is then given by

$$B = \frac{\alpha}{a} \tag{6}$$

Apportioning the Variance[2]

Suppose that a full access group is offered a random load with mean A, which is composed of the random loads with means $A_1, A_2, \ldots, A_n$, where $A = A_1 + A_2 + \ldots + A_n$. (An important property of the Poisson distribution $p(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$ is the additivity property $\{p(k;\lambda_1)\} * \{p(k;\lambda_2)\} = \{p(k;\lambda_1+\lambda_2)\}$; in other words, if two random loads are superimposed on each other, the resultant load is also random, with mean equal to the sum of the means of the component loads.) Denote the mean and variance of the overflow distribution by α and β, respectively. Let the component of the overflow resulting from A_i have mean α_i and variance β_i. Then it can be shown that

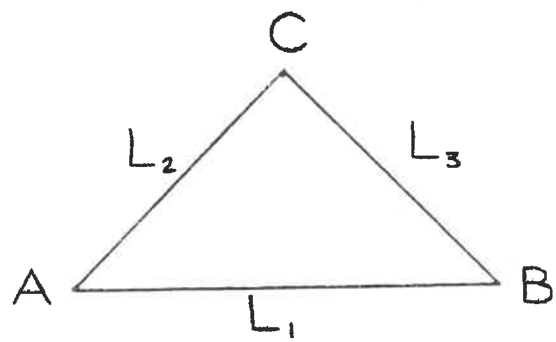
$$\alpha_i = \frac{A_i}{A} \alpha \tag{7}$$

$$\beta_i = \left(\frac{\alpha_i}{\alpha}\right)^2 \beta + \frac{\alpha_i}{\alpha} \left(1 - \frac{\alpha_i}{\alpha}\right) \alpha \tag{8}$$

Solving the Network Problem

The iteration procedure will now be described by means of an illustrative example.

Consider a network consisting of three nodes, A, B, C, and three interconnecting links, L_1, L_2, and L_3.



Each link L_i consists of c_i full access trunks, and is offered a direct random load $A_i = A_i^E + A_i^W$, where A_i^E is the eastbound and A_i^W the westbound portion of the total random load A_i. Traffic A_1^E originating at node A and bound for node B that finds all c_1 trunks busy on link L_1 tries the alternate route consisting of links L_2 and L_3; it will be lost if and only if it finds all trunks busy on L_2 and/or L_3. Similarly, A_1^W has the alternate route consisting of L_3 and L_2. L_2 and L_3 are called "finals," which means that no alternate route for overflow traffic is provided, so that traffic offered to L_2 is lost if all c_2 trunks are busy, and similarly for traffic offered to L_3.

Denote by $\alpha_i^{(n)}$ and $\beta_i^{(n)}$ the mean and variance of the load overflowing L_i at the n^{th} step of the iteration, and denote by $a_i^{(n)}$ and $v_i^{(n)}$ the mean and variance of the total load offered to L_i at the n^{th} step. Let $B_i^{(n)}$ be the blocking probability on L_i at the n^{th} step.

In order to start the iteration procedure, some initial values for the link blocking probabilities must be assumed. For each link, this can be taken as the value given by the Erlang B formula when the offered load consists of that link's first-routed random traffic only. Then the probability of blocking is $B_i^{(o)} = B(c_i, A_i)$ as given by equation (1), and the means and variances of the link overflow distributions are computed from equations (2) and (3),

$$\alpha_i^{(o)} = A_i B(c_i, A_i) \tag{9}$$

$$\beta_i^{(o)} = \left[1 - \alpha_i^{(o)} + \frac{A_i}{c_i + 1 + \alpha_i^{(o)} - A_i}\right] \alpha_i^{(o)} \tag{10}$$

The means and variances of the total loads to be offered to each of the links are now computed making use of the alternate routing pattern of the particular network under consideration.

Consider, for example, the load offered to L_2 at the n^{th} iteration step, $n = 1, 2, \ldots$. Using equation (7), the mean eastbound overflow from L_1 is $\frac{A_1^E}{A_1} \alpha_1^{(n-1)}$; the mean westbound overflow from L_1 is $\frac{A_1^W}{A_1} \alpha_1^{(n-1)}$. L_2 is not offered that part of the eastbound overflow from L_1 that encounters the condition of nonblocking on L_2 and blocking on L_3. Such traffic has negligible, if any, holding time on L_2, and its loss is not to be attributed to a blocking condition on L_2. Therefore, the mean eastbound overflow load from L_1 offered to L_2 at the n^{th} step of the iteration process is

$$\frac{A_1^E}{A_1}\,\alpha_1^{(n-1)}\left[1-B_3^{(n-1)}(1-B_2^{(n-1)})\right]$$

where $B_1^{(n-1)} = \dfrac{\alpha_1^{(n-1)}}{a_1^{(n-1)}}$ from equation (6). L_2 is also offered that part of the westbound overflow from L_1 that is not blocked on L_3; this has mean $\frac{A_1^W}{A_1}\,\alpha_1^{(n-1)}(1-B_3^{(n-1)})$. The total load offered to L_2 consists of the direct random load and the overflow from L_1. Thus, the total load offered to L_2 at step n has mean

$$a_2^{(n)} = A_2 + p_{1,2}^{(n)}\,\alpha_1^{(n-1)} \qquad (11)$$

where $p_{1,2}^{(n)}$ is the proportion of overflow traffic from L_1 that is offered to L_2 at the n^{th} step; that is

$$p_{1,2}^{(n)} = \frac{A_1^E}{A_1}\left[1-B_3^{(n-1)}(1-B_2^{(n-1)})\right] + \frac{A_1^W}{A_1}\,(1-B_3^{(n-1)}) \qquad (12)$$

(Implicit in equation (12) is the assumption that the link occupancy distributions are independent of each other.) Similarly, the total load offered to L_3 at step n has mean

$$a_3^{(n)} = A_3+p_{1,3}^{(n)}\ \alpha_1^{(n-1)} \qquad (13)$$

where

$$p_{1,3}^{(n)} = \frac{A_1^W}{A_1}\left[1-B_2^{(n-1)}(1-B_3^{(n-1)})\right]$$

$$+ \frac{A_1^E}{A_1}\ (1-B_2^{(n-1)}) \qquad (14)$$

Since L_1 receives no overflow traffic, its total offered load is simply

$$a_1^{(n)} = A_1 \qquad (15)$$

Equations (11), (12), (13), (14), and (15) give the mean values of the loads offered to the links at the beginning of the n^{th} step of the iteration, $n \geq 1$. All of the information specifying the particular alternate routing procedure of the network is contained in these equations.

The means of the nonrandom loads offered to the links have been specified; in order to compute the link blocking probabilities by the ER method, the corresponding variances must also be determined. The variances are approximated by use of equation (8).

For example, the load offered to L_2 is composed of the random component A_2 and the overflow parcels from L_1. Let $v_{1,j}^{(n)}$ be the variance associated with overflow from L_1 at step (n-1) that is offered to L_j at step n. Then applying equation (8),

$$v_{1,2}^{(n)} = (p_{1,2}^{(n)})^2 \beta_1^{(n-1)} + p_{1,2}^{(n)}(1-p_{1,2}^{(n)})\alpha_1^{(n-1)} \qquad (16)$$

The variance of the random component is numerically equal to its mean, A_2. Again assuming independence of the link occupancy distributions, (which permits neglecting any terms of covariance between link overflow distributions) the variance of the load offered to L_2 at the n^{th} iteration step is

$$v_2^{(n)} = A_2 + (p_{1,2}^{(n)})^2 \beta_1^{(n-1)} + p_{1,2}^{(n)}(1-p_{1,2}^{(n)})\alpha_1^{(n-1)} \qquad (17)$$

Similarly, the variances of the loads offered to L_2 and L_1 are given by

$$v_3^{(n)} = A_3 + (p_{1,3}^{(n)})^2 \beta_1^{(n-1)} + p_{1,3}^{(n)}(1-p_{1,3}^{(n)})\alpha_1^{(n-1)} \qquad (18)$$

and, of course,

$$v_1^{(n)} = A_1 \qquad (19)$$

Using the pairs $(a_i^{(n)}, v_i^{(n)})$, the link overflow means and variances are calculated at each step of the iteration, and the process is repeated until convergence seems assured.

It should be remarked that, in general, the use of equation (8) is only an approximation, since the offered loads at step $n \geq 1$ are not random. In the simple example given here, $\alpha_1^{(n)} = \alpha_1^{(o)}$ and $\beta_1^{(n)} = \beta_1^{(o)}$, since L_1 receives only the random load A_1. Therefore, equations (17) and (18) are, in this case, correct expressions for the variance associated with the proportion $p_{i,j}^{(n)}$ of the overflow traffic.

It is now simple though tedious to carry out the iteration procedure until the $B_i^{(n)} = \alpha_i^{(n)}/a_i^{(n)}$ converge to the link blocking probabilities, B_i. Note that each B_i represents an average link blocking probability, and not, in general, the link blocking as seen by any particular parcel of traffic. Since overflow calls tend to occur in bunches, it appears that overflow traffic experiences poorer service on L_i, and direct traffic better service on L_i, than is indicated by the over-all link blocking probability, B_i. Note also that the B_i are link losses, not point-to-point losses. The calculation of the latter quantities is discussed in the appendices of [3].

A theoretical investigation of the question of convergence of the iteration process is difficult. Even when the process does converge, it is possible a priori that the values to which it converges depend upon the initial values assumed for the link blocking probabilities. If this is suspected, the process can be repeated a second time, using as initial values the final values of the preceding iteration. All cases studied so far have converged quickly. Dependence upon initial conditions, if it exists at all, appears to be insignificant.

It should be noted that the example presented above is a very simple one compared to most practical networks. In fact, in this example, $\beta_2^{(n)}$ and $\beta_3^{(n)}$ need never be calculated, and $\alpha_1^{(n)} = \alpha_1^{(o)}$, $\beta_1^{(n)} = \beta_1^{(o)}$, and $B_1^{(n)} = B_1^{(o)}$ are the same for all values of n. Especially simple in form are the equations for $a_1^{(n)}$ and $v_1^{(n)}$, which become unwieldy as the network size and complexity increase. (See Appendix for equations representing an actual network.)

An Evaluation of the Nonrandom Model

1. The New England Network Study

The iteration procedure described in the previous section has been used to evaluate the grade of service encountered in a 7-node hierarchical network for a variety of traffic conditions under two different alternate routing plans.[3] The network employed in this study (sometimes called "The New England Network") is identified in Figure 1. Two alternate routing plans for this network are presented in Table 1.

To illustrate the iteration equations necessary for the solution of the two-parameter (nonrandom) traffic model representing the traffic flow in the 7-node network, a sample of the equations corresponding to routing plan I is presented in Appendix A. For each trunk group in the network, the mean and variance of the total offered load are expressed in terms of the segments of direct routed traffic that are ultimately offered to the given link. The traffic offered to trunk groups 1 to 4, as indicated in the appendix, is random since no overflow calls are routed to these links. Both direct and alternate routed traffic are offered to each of the other trunk groups in the network.

The equations derived for the nonrandom traffic model were utilized to compute the average blocking probability for each trunk group in the network under a variety of traffic loads.

2. Comparison of the Nonrandom Iteration Method and Simulation Techniques

The accuracy of the link blocking estimates obtained from the iteration procedure, based upon the two-parameter (nonrandom) traffic model was examined by comparing them with the blocking probabilities obtained by simulation. A comparison of the blocking probabilities is presented in Table 2 for normal engineered loads and in Table 3 for a typical overload condition. An examination of Tables 2 and 3 indicates that the link blocking estimates computed by the iteration procedure using the two-parameter description of overflow traffic and those obtained by simulation techniques are in reasonable agreement. (Some idea of the accuracy of the simulation results is evidenced by a comparison with the computed figures for trunk groups 1 through 4. Since these trunk groups receive no overflow traffic, the computed results are theoretically exact.)

The results obtained analytically were computed in a small fraction of the time required for a simulation run giving acceptable accuracy. The iteration procedure is therefore particularly useful in analyzing alternate routing networks when a number of different traffic conditions are to be investigated.

3. Comparison of Random and Nonrandom Models

In order to demonstrate the improvement in accuracy achieved by considering the nonrandomness of overflow traffic, a simple one-parameter model was also employed in analyzing the 7-node hierarchical network. For this model, the mean of the probability distribution governing the load offered to each trunk group in the network is assumed sufficient to characterize traffic flow. This corresponds to the assumption that all traffic, direct and overflow, is random.

The average loads offered to the trunk groups are still represented by the same equations ((11), (12), (13), (14), and (15)), but the corresponding variances are not specified. The ER method and the variance apportioning procedure are no longer used; the $B_i^{(m)}$ are now given directly by $B_i^{(m)} = B(c_i, a_i^{(m)})$. The blocking probabilities obtained by means of this random model are presented in Tables 2 and 3.

A comparison of link blocking estimates obtained from the random model with those obtained from the more complicated two-parameter traffic model indicates that for normal engineered loads a decided loss of accuracy may result from ignoring the nonrandomness of overflow traffic. For trunk groups to which predominantly alternate routed traffic is offered, the iteration solution of the random model appears to underestimate the actual link blocking probabilities. This is illustrated in Table 2 for trunk groups R and S of the New England network.

Thus, the simplicity of the iteration procedure obtained by neglecting the nonrandomness of overflow traffic is achieved at the expense of the accuracy of the link blocking estimates. The present example suggests that application of the random model for network analysis at engineered load levels should be limited to cases in which there is little overflow traffic to deal with, or to applications in which only rough approximations are required.

An examination of Table 3 indicates that better accuracy is attained with the random model for overload traffic conditions in which the link blocking probabilities are fairly sizable. This suggests that the one-parameter model may, on occasion, be suitably applied for network evaluations under overload conditions.

4. Point-to-Point Loss Calculations

An important aspect of alternate routing network analysis is the calculation of the point-to-point loss probability between each pair of nodes in the network. These values may be approximated by properly combining the average link blocking probabilities (Appendix A, reference [3]).

A comparison of analytic and simulation estimates of the point-to-point loss probabilities encountered in the New England network at engineered loads is given in Table 4. The analytic estimates are computed from the link blocking probabilities obtained by the iteration procedure using the nonrandom traffic model.

As noted previously, the blocking probability for each trunk group is the average value, not necessarily the blocking probability as seen by each parcel of traffic offered to that link. For trunk groups such as R, receiving predominantly alternate routed traffic, the blocking probability encountered by the direct routed (random) traffic is substantially less than the average link blocking value. Hence, the actual values of the point-to-point loss probabilities between offices terminating these trunk groups are appreciably less than the analytic estimates. An illustration of this is found in Table 4 where the loss probability for direct routed traffic on trunk group R is less than half of the average link blocking probability. For most of the other office pairs, the analytic estimates of loss probability underestimate the actual (simulation) values.

As might be expected, the point-to-point loss estimates are not as accurate as the link blocking values; this appears to be the result of the assumption of independence of link occupancy distributions and the use of the average link blocking probability for all traffic parcels offered to a given trunk group. However, for initial loss estimation, or for comparison studies, the analytically obtained point-to-point losses may be quite useful.

Concluding Remarks

The method presented here for the analysis of alternate routing communications networks appears to have a number of valuable uses. For example, in a moderately complicated network arrangement, the service expected on each trunk group can be estimated with good reliability once the offered loads are specified. From these link blocking estimates the point-to-point service can also be approximated. The method lends itself particularly well to an investigation of the effects of load variations on a given network, since the appropriate calculations are performed quickly once the network equations have been programed.

It is evident, however, from the New England network example, that the derivation of the equations necessary for the analysis of practical networks is a laborious task to perform manually. Since the rules governing the derivation of the iteration equations are conceptually simple, it would appear that a computer program could be written to generate these equations as well as solve them.

A computer program of this type has been developed by G. R. Faulhaber. Unfortunately, this program also appears to be restricted to communications networks of limited size, since the number and complexity of equations required for the analysis grows quite rapidly with the size of the network. A detailed description of this program will be presented in a forthcoming memorandum by Mr. Faulhaber.

In order to cope with larger alternate routing networks, an analytic procedure has been developed by S. S. Katz, to compute link blocking and point-to-point loss probabilities, using the two-parameter description of traffic, without having to generate the iteration equations. In addition, the procedure is expected to improve the point-to-point service calculations. The method is currently under investigation.

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Att.
Figure 1
Tables 1-4
References
Appendix

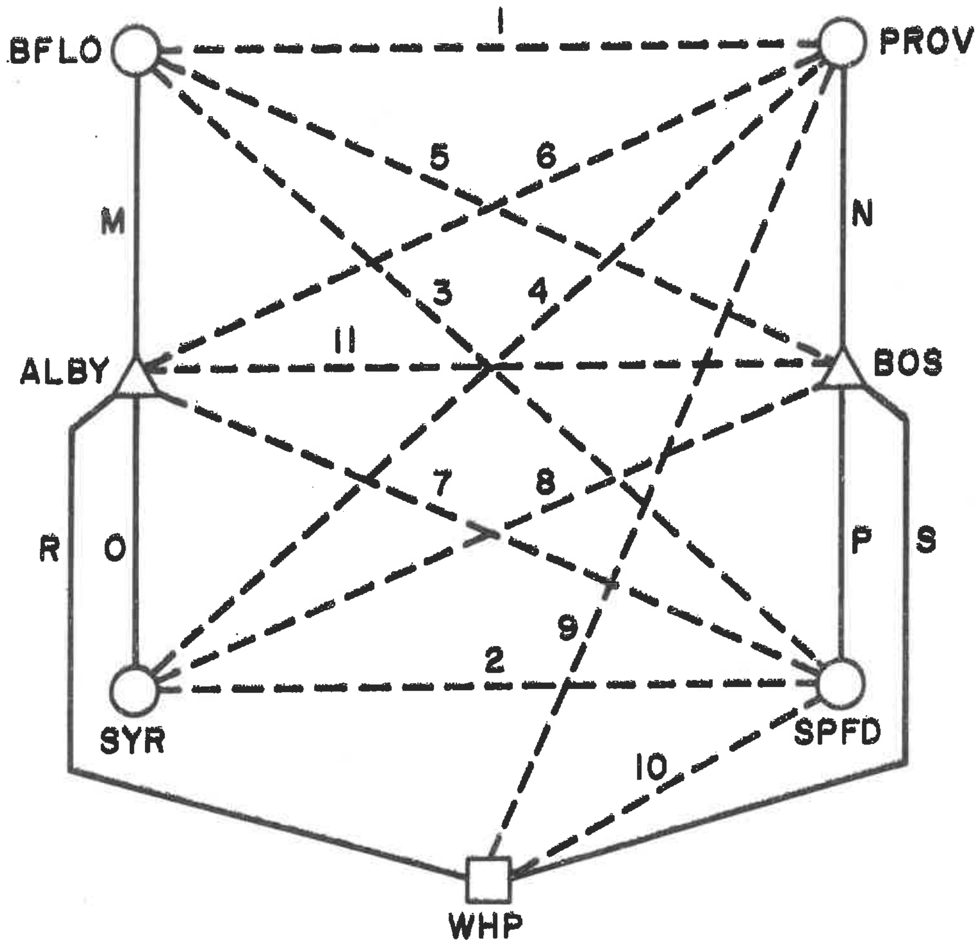

FIG. I

HIERARCHICAL ALTERNATE ROUTING NETWORK

TABLE 1

ROUTING PATTERNS AND TRUNK REQUIREMENTS
FOR THE INTERTOLL TRUNK NETWORK

			PLAN I			PLAN II				
Trunk Group	Group Terminals		Nondirectional Alternate Route		No. of Trunks	Alt. Rt. for Directional Traffic Eastbound		Westbound		No. of Trunk
1	BFLO	PROV	M	6	11	5	N	6	M	11
2	SPFD	SYR	P	8	17	8	P	7	O	17
3	BFLO	SPFD	M	7	12	5	P	7	M	12
4	PROV	SYR	N	8	16	8	N	6	O	16
5	BFLO	BOS	M	11	30	Same as PLAN I				32
6	PROV	ALBY	N	11	14					14
7	SPFD	ALBY	P	11	24					24
8	SYR	BOS	O	11	47					46
9	PROV	WHP	N	S	44					44
10	SPFD	WHP	P	S	39					39
11	ALBY	BOS	R	S	85					84
M	BFLO	ALBY	-	-	56					54
N	PROV	BOS	-	-	81					81
O	SYR	ALBY	-	-	92					93
P	SPFD	BOS	-	-	69					69
R	ALBY	WHP	-	-	28					29
S	BOS	WHP	-	-	98					98

Table 2

A Comparison of Link Blocking Probabilities
Computed by Simulation and Iteration Models*

Case 1: Engineered Load Levels

Trunk Group Identity	Analytic Estimates		Simulation Estimates
	Random Model	Nonrandom Model	
1	0.151	0.151	0.138
2	0.113	0.113	0.112
3	0.120	0.120	0.103
4	0.019	0.019	0.017
5	0.164	0.167	0.161
6	0.115	0.116	0.114
7	0.084	0.084	0.083
8	0.172	0.174	0.179
9	0.090	0.090	0.091
10	0.048	0.048	0.048
11	0.061	0.076	0.077
M	0.002	0.006	0.009
N	0.012	0.016	0.019
O	0.002	0.005	0.008
P	0.007	0.009	0.009
R	0.013	0.071	0.085
S	0.019	0.033	0.038

*Alternate routing Plan I was employed for this comparison.

Table 3

A Comparison of Link Blocking Probabilities
Computed by Simulation and Iteration Models*

Case 2: 20% Overload Conditions

Trunk Group Identity	Analytic Estimates: Random Model	Analytic Estimates: Nonrandom Model	Simulation Estimates
1	0.206	0.206	0.199
2	0.171	0.171	0.171
3	0.225	0.225	0.231
4	0.206	0.206	0.196
5	0.296	0.296	0.290
6	0.237	0.245	0.242
7	0.167	0.172	0.173
8	0.285	0.288	0.270
9	0.169	0.169	0.170
10	0.145	0.145	0.147
11	0.230	0.240	0.240
M	0.094	0.104	0.098
N	0.073	0.081	0.088
O	0.102	0.110	0.116
P	0.070	0.080	0.081
R	0.341	0.393	0.379
S	0.186	0.195	0.192

*Alternate routing Plan I was employed for this comparison.

Table 4

A Comparison of Analytic and Simulation Estimates of Point-to-Point Loss Probability*

Trunk Group Identity	Analytic Estimates (From Nonrandom Model)	Simulation Estimates
1	0.002	0.003
2	0.001	0.002
3	0.001	0.001
4	0.0002	0.0002
5	0.002	0.008
6	0.003	0.005
7	0.001	0.003
8	0.002	0.008
9	0.004	0.007
10	0.002	0.003
11	0.008	0.016
M	0.006	0.005
N	0.016	0.016
O	0.005	0.005
P	0.009	0.007
R	0.071	0.034
S	0.033	0.026

*Engineered load levels and alternate routing Plan I were used for this comparison.

REFERENCES

1. Wilkinson, R. I., Theories for Toll Traffic Engineering in the U.S.A., Bell System Technical Journal, 35, 421-514, 1956.

2. Descloux, A., On the Components of Overflow Traffic, MM-62-3122-9.

3. Katz, S. S., Directional Alternate Routing in Intertoll Network, MM-64-3122-1.

APPENDIX

Equations Employed in the Link Blocking Analysis of the New England Network

The equations presented here define the mean and variance of the total load offered to some of the trunk groups of the New England network for the alternate routing pattern identified as Plan I (see Table 1).

Trunk Groups 1-4

For $i = 1,2,3,4$

$$a_i = A_i$$

$$v_i = A_i$$

Trunk Group 6

Let $$p_1 = \frac{1}{a_1} \{A_1^E(1-B_M) + A_1^W[1-B_M(1-B_6)]\}$$

$$p_2 = \frac{1}{a_8} \{A_4^E B_4[1-B_N(1-B_8)](1-B_0)\}$$

Then, $$a_6 = A_6 + p_1\alpha_1 + p_2\alpha_8$$

$$v_6 = A_6 + (p_1)^2\beta_1 + p_1(1-p_1)\alpha_1 + (p_2)^2\beta_8 + p_2(1-p_2)\alpha_8$$

Trunk Group 11

Let $p_1 = \frac{1}{a_6} \{A_1^E B_1 (1-B_M) + A_4^E B_4 B_8 [1-B_N(1-B_8)](1-B_O)$

$+ A_6^E\} \cdot [1-B_N(1-B_{11})]$

$p_2 = \frac{1}{a_6} \{A_6^W (1-B_N)\}$

$p_3 = \frac{1}{a_5} \{A_1^W B_1 B_6 [1-B_M(1-B_M(1-B_6)](1-B_N)$

$+ A_3^W B_3 B_7 [1-B_M(1-B_7)](1-B_p) + A_5^W\} [1-B_M(1-B_{11})]$

$p_4 = \frac{1}{a_5} \{A_5^E (1-B_M)\}$

$p_5 = \frac{1}{a_7} \{A_2^E B_2 B_8 [1-B_p(1-B_8)](1-B_O) + A_7^E$

$+ A_3^E B_3 (1-B_M)\} [1-B_p(1-B_{11})]$

$p_6 = \frac{1}{a_7} \{A_7^W (1-B_p)\}$

$p_7 = \frac{1}{a_8} \{A_2^W B_2 (1-B_p) + A_4^W B_4 (1-B_N) + A_8^W\} \cdot [1-B_O(1-B_{11})]$

$p_8 = \frac{1}{a_8} \{A_8^E (1-B_O)\}$

Then, $a_{11} = A_{11} + (p_1+p_2)\alpha_6 + (p_3+p_4)\alpha_5$

$(p_5+p_6)\alpha_7 + (p_7+p_8)\alpha_8$

$$
\begin{aligned}
v_{11} = A_{11} &+ (p_1+p_2)^2\beta_6 + (p_1+p_2)(1-p_1-p_2)\alpha_6 \\
&+ (p_3+p_4)^2\beta_5 + (p_3+p_4)(1-p_3-p_4)\cdot\alpha_5 \\
&+ (p_5+p_6)^2\beta_7 + (p_5+p_6)(1-p_5-p_6)\cdot\alpha_7 \\
&+ (p_7+p_8)^2\beta_8 + (p_7+p_8)(1-p_7-p_8)\cdot\alpha_8
\end{aligned}
$$

Trunk Group P

Let

$$x_1 = (1-B_p)B_8(B_{11}+B_O-B_{11}\cdot B_O)[1-(1-B_S)(1-B_R)(1-B_O)]$$

$$x_2 = (1-B_p)B_5(B_{11}+B_M-B_{11}\cdot B_M)[1-(1-B_S)(1-B_R)(1-B_M)]$$

$$x_3 = (1-B_p)B_{11}(B_S+B_R-B_SB_R)$$

In addition, we define:

$$p_1 = \frac{1}{a_7}\{A_2^E B_2 B_8[1-B_p(1-B_8)](1-B_O) + A_7^E + A_3^E B_3(1-B_M)\}(1-B_{11})$$

$$p_2 = \frac{1}{a_7}\{A_3^W B_3[1-B_M(1-B_7)]\}[1-x_2]$$

$$p_3 = \frac{1}{a_7}\{A_7^W[1-x_3]\}$$

$$p_4 = \frac{1}{a_{10}}\{A_{10}^W[1-B_S(1-B_p)] + A_{10}^E(1-B_S)\}$$

$$p_5 = \frac{1}{a_{10}}\{\frac{p_1}{1-B_{11}}\alpha_7 B_{11}[1-B_p(1-B_{11})](1-B_R)(1-B_S)\}$$

$$p_6 = \frac{1}{a_2}\{A_2^W[1-x_1] + A_2^E(1-B_8)\}$$

Then,

$$a_p = A_p + (p_1+p_2+p_3)\alpha_7 + (p_4+p_5)\alpha_{10} + p_6\alpha_2$$

$$v_p = A_p + (p_1+p_2+p_3)^2\beta_7 + (p_1+p_2+p_3)(1-p_1-p_2-p_3)\alpha_7$$

$$+ (p_4+p_5)^2\beta_{10} + (p_4+p_5)(1-p_4-p_5)\alpha_{10}$$

$$+ (p_6)^2\beta_2 + p_6(1-p_6)\alpha_2$$