Distributed Dominant Pruning in Ad Hoc Networks

Fei Dai and Jie Wu
Department of Computer Science and Engineering
Florida Atlantic University, Boca Raton, FL 33431

***Abstract*— Efficient routing among mobile hosts is an important function in ad hoc networks. Routing based on a connected dominating set is a promising approach, where the search space for a route is reduced to the hosts in the set. A set is dominating if all the hosts are either in the set or neighbors of hosts in the set. The efficiency of dominating-set-based routing mainly depends on the overhead introduced in the formation of the dominating set and the size of the dominating set. In this paper, we first review a distributed formation of a connected dominating set called *marking process* and dominating-set-based routing. Then we propose a dominant pruning rule to reduce the size of the dominating set. This dominant pruning rule (called Rule k) is a generalization of two existing rules (called Rules 1 and 2). We prove that the vertex set derived by applying Rule k is still a connected dominating set. When implemented with local neighborhood information, Rule k is more effective in reducing the dominating set derived from the marking process than the combination of Rules 1 and 2, and has the same communication complexity and less computation complexity. Simulation results confirm that Rule k outperforms Rules 1 and 2, especially in relatively dense networks with unidirectional links.**

I. Introduction

An ad hoc network can be represented by a *unit disk graph* [1], where every vertex (host) is associated with a disk centered at this vertex with the same radius (transmitter range). Two vertices are neighbors if and only if they are covered by each other's disk. For example, both vertices v and w in Figure 1 (a) are neighbors of vertex u because they are covered by disk u; while vertices v and x in Figure 1 (b) are not neighbors. In an ad hoc network, some links (edges) may be unidirectional due to either the disparity of energy levels of hosts or the hidden terminal problem [2]. Therefore, a general ad hoc network can be considered as a directed graph with a high percentage of bidirectional links.

Routing in ad hoc networks is harder than that in wired networks, due to the limited resource, network mobility and lack of a physical infrastructure. Virtual infrastructures, such as a *connected dominating set* (CDS), were proposed to reduce the routing overhead and enhance scalability. A dominating set satisfies that every vertex in the graph is either in the set or adjacent to a vertex in the set. Vertices in a dominating set are also called *gateways* while vertices that are outside a dominating set are called *non-gateways*. Among CDS-based routing protocols, only gateways need to keep routing information in a *proactive approach* and the search space is reduced to the dominating set in a *reactive approach*. Clearly, the efficiency of this approach depends largely on the process of finding and maintaining a CDS and the size of the corresponding subnetwork.

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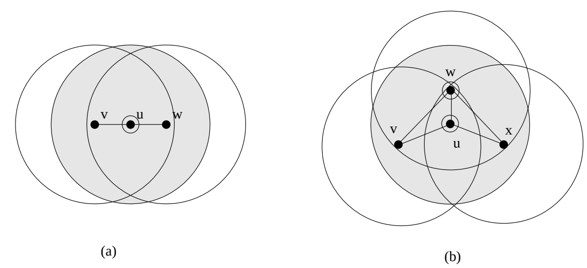

Fig. 1. Examples of ad hoc networks.

Unfortunately, finding a minimum connected dominating set is NP-complete for most graphs. Wu and Li proposed a simple and efficient approach called *marking process* which can quickly determine a CDS. This approach was first proposed for undirected graphs using the notion of dominating set only [3] and was later extended to cover directed graphs by introducing another notion called *absorbent set* [4]. Specifically, each host is *marked* (i.e., becomes a gateway) if it has two unconnected neighbors. It is shown that collectively these hosts achieve a desired global objective – a set of marked hosts forms a small CDS. Based on the marking process, vertices u and w in Figure 1 (b) are marked and they form a dominating set in their network. The CDS derived from the marking process is further reduced by applying two *dominant pruning rules*. According to dominant pruning Rule 1, a marked host can unmark itself if its neighbor set is covered by another marked host; that is, if all its neighbors are connected with each other via another gateway, it can relinquish its responsibility as a gateway. In Figure 1 (b), either u or w can be unmarked (but not both). According to Rule 2, a marked host can unmark itself if its neighborhood is covered by two other directly connected marked hosts. The marking process and Rules 1 and 2 are purely localized algorithms where the marker of a host depends on topology of a small vicinity only.

We propose a generic dominant pruning rule called Rule k, which can unmark gateways covered by k other gateways, where k can be any number. Rule k is more efficient in reducing the number of gateways than the combination of Rules 1 and 2. For example, if hosts in Figure 1 are evenly distributed and dense enough, it is almost impossible to find two hosts v and w to cover the neighborhood of host u (see the shadowed area in Figure 1 (a)). However, it is much easier to find three or more hosts to cover the same neighborhood (see Figure 1 (b)). An efficient algorithm is developed that implements a restricted version of Rule k with less computation complexity than that of the combination of Rules 1 and 2. Simulation results of this paper show that this restricted version of Rule k outperforms the combination of the restricted Rules 1 and 2 in reducing the number of gateways.

II. Related Work

Das et al. [5] proposed an algorithm to identify a subnetwork that forms a *minimum CDS* (MCDS). This algorithm finds a CDS by growing a tree T starting from a vertex with the maximum vertex degree, and adding new vertices to T according to its effective degree (number of neighbors that are not neighbors of T). The main drawback of this algorithm is its centralized style: Vertices in the MCDS are selected sequentially, and expensive coordination is needed for each selection unless global information is provided.

Several algorithms were proposed based on clusters. A cluster usually contains a *head* and several *members* that are neighbors of the head. Lin and Gerla [6] gave two simple clustering algorithms based on host id and degree, respectively. The clustering approaches are very effective in reducing the size of the dominating set in very dense networks. However, the head election process may have to be serialized in some cases, such as in a linear network with monotonically increasing or decreasing id distribution along the network.

Heads of clusters form a dominating set, but they are not necessarily connected. Some *border members* (i.e., members with neighbors in other clusters) are designated as gateways, which form virtual links between cluster heads and connect all clusters. In the *maximum connectivity* scheme, most border members are designated as gateways. The objective here is to maximize the throughput and reliability, rather than to reduce the size of a connected dominating set. The *mesh* scheme [7] designates a subset of border members as gateways so that there is exactly one virtual link between two neighboring clusters. The *tree* scheme [8] minimizes the number of virtual links by growing a breadth-first search tree via flooding. However, this approach demands a root election, which is quite expensive, and the flooding needs $O(\delta)$ rounds to complete, where δ is the diameter of the network.

Wu and Li's marking process uses a constant number of rounds to determine a CDS. This approach can be applied to ad hoc networks with bidirectional links only [3] and with both bidirectional and unidirectional links [4]. Stojmenovic et al. [9] further reduced the communication overhead of the localized dominating set algorithm using location information.

III. Preliminaries

An ad hoc network with unidirectional links can be represented by a simple directed graph $G = (V, E)$, where V is a set of vertices (hosts) and E is a set of directed edges (unidirectional links). A directed edge from u to v is denoted by an ordered pair (u, v). A directed graph G is strongly connected if for any two vertices u and v, a (u, v)-path (i.e., a path connecting u to v) exists. If (u, v) is an edge in G, we say that u dominates v, and v is an absorbent of u. The *dominating neighbor set* $N_d(u)$ of vertex u is defined as $\{w : (w, u) \in E\}$. The *absorbent neighbor set* $N_a(u)$ of vertex u is defined as $\{v : (u, v) \in E\}$. $N(u) = N_d(u) \cup N_a(u)$ represents the neighbor set of vertex u.

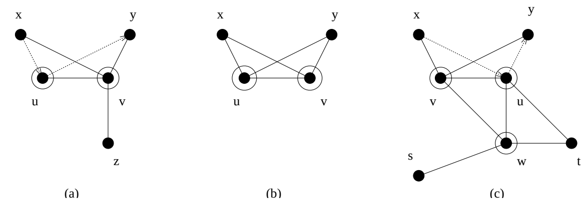

Fig. 2. Three examples of dominating set reduction.

A set $V' \subset V$ is a *dominating set* of G if every vertex $v \in V - V'$ is dominated by at least one vertex $u \in V'$. Also, a set $V' \subset V$ is called an *absorbent set* if for every vertex $u \in V - V'$, there exists a vertex $v \in V'$ which is an absorbent of u. For example, vertex set $\{u, v\}$ in Figures 2 (a) and (b) and $\{u, v, w\}$ in Figure 2 (c) are both dominating and absorbent sets of the corresponding directed graphs. The absorbent subset may overlap with the dominating subset. We use the term "(connected) dominating set" to represent "(strongly connected) dominating and absorbent set". The following marking process can quickly find a strongly connected dominating and absorbent set in a given directed graph. All nodes are initially marked F (unmarked).

Algorithm 1 Marking process [4]

1: Each u periodically exchanges its neighbor set $N_d(u)$ and $N_a(u)$ with all its neighbors.
2: u sets its marker to T (marked) if there exist two neighbors v and w of u such that $(w, u) \in E$, $(u, v) \in E$ and $(w, v) \notin E$.

Suppose the marking process is applied to the network represented by Figure 2 (a). Host u will be marked because $(x, u) \in E$ and $(u, y) \in E$, but $(x, y) \notin E$; host v will also be marked. All other hosts will remain unmarked because no such pair of neighbor hosts can be found. Results in [4] show that marked vertices form a strongly connected dominating and absorbent set, and furthermore, can connect any two vertices with minimum hops.

Two dominant pruning rules are proposed in [3] and then extended in [4] to reduce the size of the CDS derived from the marking process. We say a vertex is *covered* if its neighbors can reach each other via other connected marked vertices. If a vertex is covered by no more than two connected vertices, removing this vertex from V' will not compromise its functionality as a CDS. To avoid simultaneous removal of two vertices covering each other, each vertex $v \in V$ is assigned a distinct id, $id(v)$. A vertex is removed only when it is covered by vertices with higher id's.

Rule 1: *Consider two vertices u and v in G' (induced subgraph of V'). If $N_d(u) - \{v\} \subseteq N_d(v)$ and $N_a(u) - \{v\} \subseteq N_a(v)$ in G and $id(u) < id(v)$, change the marker of u to F.*

Rule 2: *Assume that v and w are bidirectionally connected in G'. If $N_d(u) - \{v, w\} \subseteq N_d(v) \cup N_d(w)$ and $N_a(u) - \{v, w\} \subseteq$*

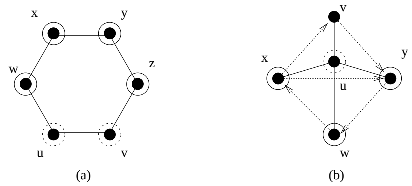

Fig. 3. Limitation of Rules 1 and 2.

$N_a(v) \cup N_a(w)$ *in* G *and* $id(u) = \min\{id(u), id(v), id(w)\}$, *then change the marker of* u *to* F.

In Figure 2 (a), since $N_d(u) - \{v\} \subseteq N_d(v)$, $N_a(u) - \{v\} \subseteq N_a(v)$ and $id(u) < id(v)$, vertex u is removed from V'. In Figure 2 (b), u and v cover each other, but only u is removed from V' because $id(u) < id(v)$. In Figure 2 (c), since $N_d(u) - \{v, w\} \subseteq N_d(v) \cup N_d(w)$, $N_a(u) - \{v, w\} \subseteq N_a(v) \cup N_a(w)$, and $id(u) = \min\{id(u), id(v), id(w)\}$, vertex u can be removed from V' based on Rule 2. It is proved in [4] that the reduced set $V'_* \subseteq V'$ generated from applying Rule 1 and/or Rule 2 to V' is still a strongly connected dominating and absorbent set of G. The combination of the marking process and Rules 1 and 2 is a purely localized algorithms.

IV. Pruning Through k-Neighbor Coverage

A. Generic pruning rule

Let $G' = (V', E')$ be the induced subgraph of G by V', $V'_k = \{v_1, v_2, ..., v_k\}$ is the vertex set of a strongly connected subgraph in G', $N_d(V'_k) = \bigcup_{v_i \in V'_k} N_d(v_i)$, and $N_a(V'_k) = \bigcup_{v_i \in V'_k} N_a(v_i)$.

Rule k: *If* $N_d(u) - V'_k \subseteq N_d(V'_k)$ *and* $N_a(u) - V'_k \subseteq N_a(V'_k)$ *in* G *and* $id(u) = \min\{id(u), id(v_1), id(v_2), \cdots, id(v_k)\}$, *then change the marker of* u *to* F.

Rules 1 and 2 are the special cases of Rule k, where $|V'_k|$ is restricted to 1 and 2, respectively. However, a vertex removed by Rule k is not necessarily removable by Rule 1 or Rule 2. For example, in Figure 3 (a), both vertices u and v can be removed by Rule k (for $k \geq 3$) because they are covered by vertices w, x, y, and z; in Figure 3 (b), vertex u can be removed because it is covered by vertices w, x, and y. Note that although x and y are not bidirectionally connected directly, they can reach each other via vertex w. However, none of these vertices can be removed by Rule 1 or Rule 2, because they cannot be covered by one or two bidirectionally directly connected vertices.

Theorem 1: If V' is a strongly connected dominating and absorbent set of G, and V'_R is the set of vertices removable under Rule k, then $V'_* = V' - V'_R$ is a strongly connected dominating and absorbent set of G.

Proof: : First we prove that V'_* is a dominating set of G. This claim holds when $|V'| = 1$, because $V'_* = V'$. If $|V'| > 1$, for every vertex u in G, it is either in V' or not in V'. If $u \notin V'$, it is dominated by at least one vertex in V', because V' is a dominating set of G. If $u \in V'$, it is also dominated by a vertex in V', because V' is strongly connected. In addition, there always exists a vertex $v \in V'$ satisfying $id(v) = \max\{id(w) : w \in N_d(u)\}$, which cannot be removed by applying Rule k. Therefore, u is dominated by at least one vertex $v \in V'_*$. By analogy we can prove that V'_* is aa absorbent set of G.

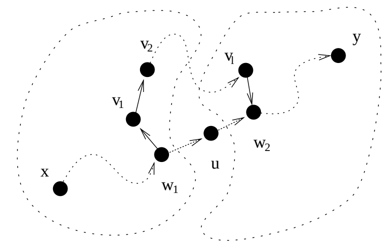

Fig. 4. An impossible network partition.

Next we prove that $G[V'_*]$ is strongly connected. Suppose that $G[V'_*]$ is not strongly connected. If we put back the removed vertices one by one in the descending order of vertex id's, we shall find the first vertex u that "re-connects" V'_*; that is, after the removal of u, at least one pair of vertices (x, y) in $G[V']$ loses its last connecting path. However, this is impossible: If u is removed from V' by applying Rule k, its dominating and absorbent neighbor sets are covered by a strongly connected set of vertices with higher id's than $id(u)$. As we can see in Figure 4, for any (x, y)-path through u, there always exists another (x, y)-path with the following three segments: (1) from source x to vertex w_1 before u, (2) from w_1 to the vertex after u, w_2, through $v_1, v_2, \cdots, v_l$ covering u, and (3) from w_2 to destination, which is not through u. Therefore, removal of u cannot eliminate all (x, y)-paths, which is a contradiction. ■

B. An efficient pruning algorithm

There are two ways to implement a dominant pruning rule: *restricted* or *non-restricted*. In the restricted implementation, a host unmarks itself only when it is covered by a group of self-connected marked neighbors. In the non-restricted implementation, a host can be covered by a group of hosts 1 or 2 hops away, self-connected or connected by other marked hosts. For example, hosts u and v in Figure 3 (a) and u in Figure 3 (b) can be unmarked by the non-restricted Rule k, but only host u in Figure 3 (b) can be unmarked by the restricted Rule k. Hosts u and v in Figure 3 (a) cannot unmark themselves because one of the covering hosts, w, is not a neighbor of them. Simulation results show that in average ad hoc networks, the number of hosts unmarked by restricted and non-restricted rules are very close. From the practicality of implementation, the restricted implementation is much better because it only needs 2-hop neighborhood information.

In the restricted k-dominant pruning algorithm, each host decomposes the induced graph of its marked neighbor set with higher id's V'_+ into several *strong components*. The strong components [10] of a directed graph are the equivalence classes of vertices under the "mutually reachable" relation. Two vertices of V'_+ belong to the same strong component if and only if they are strongly connected in $G[V'_+]$. For

Algorithm 2 Restricted k-dominant pruning (at each $u \in V'$)

1: Send a notification packet to each neighbor v satisfying $id(v) < id(u)$.
2: Receive all notification packets and build a subgraph $G[V_+^{'}]$, where $V_+^{'} = \{w | w \in (V^{'} \cap N(u)) \wedge (id(u) < id(w))\}$ is u's marked neighbor set with higher id's.
3: Compute the set of strongly connected components $\{V_{c_1}^{'}, V_{c_2}^{'}, \cdots, V_{c_l}^{'}\}$ of $G[V_+^{'}]$.
4: Change its marker $m(u)$ to F if there exists $V_{c_i}^{'}, 1 \leq i \leq l$, such that $N_d(u) - V_{c_i}^{'} \subseteq N_d(V_{c_i}^{'})$ and $N_a(u) - V_{c_i}^{'} \subseteq N_a(V_{c_i}^{'})$.

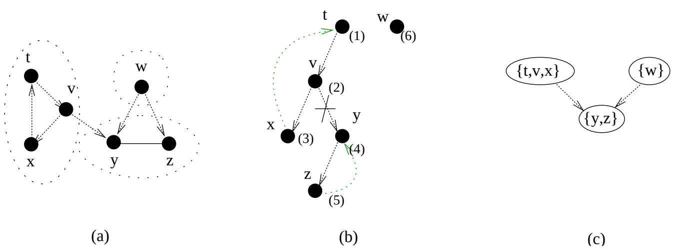

Fig. 5. Decomposition of strong components.

example, the directed graph in Figure 5 has three strong components: $\{t, v, x\}$, $\{w\}$, and $\{y, z\}$. A directed graph is strongly connected if it has only one strong component. Note that although we always assume that G' is a strongly connected graph, $G[V_+^{'}]$ is not necessarily strongly connected. For any marked host u, if it can be unmarked by applying the restricted Rule k, it must be covered by a subset of a strong component, $V_{c_i}^{'}(1 \leq i \leq l)$, which also covers u. If u is not covered by any $V_{c_i}^{'}$, it cannot be covered by any other strongly connected vertex set. Therefore, it is not necessary to test the coverage of every combination of u's marked neighbors: *Testing every strongly connected component shall be sufficient.*

Several linear-time algorithms have been developed to decompose a directed graph $G = (V, E)$ into strong components [10], [11]. They are all based on the *depth-first search* (DFS) algorithm and have a complexity of $O(|E| + |V|)$. Details of adopting these algorithms in the restricted k-dominant pruning can be found in [12].

C. Complexity analysis

The following theorems show that the restricted Rule k has the same communication complexity as restricted Rules 1 and 2, the same computation complexity as restricted Rule 1, and less computation complexity than restricted Rule 2. Their proofs can be found in [12].

Theorem 2: The marking process and the restricted versions of Rules 1, 2 and k have the same communication complexity $O(\Delta)$, where Δ is the maximum vertex degree in the network.

Theorem 3: The marking process and the restricted versions of Rule 1 and Rule k have the same computation complexity $O(\Delta^2)$. The computation complexity of the restricted Rule 2 is $O(\Delta^3)$.

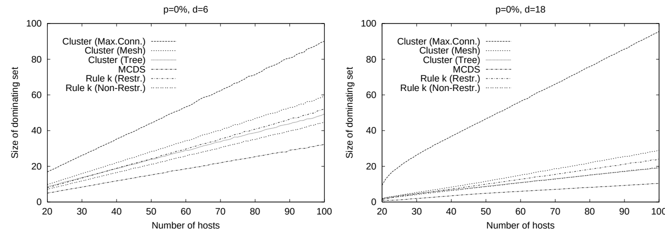

Fig. 6. Rule k vs. cluster-based schemes and MCDS.

V. SIMULATION

The simulation is conducted by our dominating set algorithm testbed *ds*, which simulates several connected dominating set algorithms, including the marking process and several dominant pruning rules (Rules 1, 2, and k), MCDS, and three cluster-based schemes (maximum connectivity, mesh, and tree). To generate a random ad hoc network, n hosts are randomly placed in a restricted 100×100 area. The transmitter range r is adjusted according to the average vertex degree d to produce exactly $\frac{nd}{2}$ links in the corresponding unit disk graph. Most of these links are treated as bidirectional, but a small portion ($p\%$) of them are randomly selected to be unidirectional links. Networks that cannot form a strongly connected graph are discarded. For each combination of parameters (n, d, and p), the simulation is repeated 500-2000 times until the confidence interval is sufficiently small ($\pm 1\%$, for the confidence level of 90%).

Figure 6 compares the performance of Rule k, in terms of the sizes of resultant dominating sets, with MCDS and three cluster-based schemes on two types of undirected graphs: relatively sparse ones (the left graph, $d = 6$) and relatively dense ones (the right graph, $d = 18$). Among these algorithms, MCDS is the best (i.e., produces the smallest CDS) and the maximum connectivity scheme is the worst. The performance of Rule k and the other two clustering schemes (tree and mesh) lies between them. The performance from the worst to the best is: the mesh scheme, the restricted Rule k, the non-restricted Rule k, and the tree scheme. The difference between the mesh scheme, Rule k, and the tree scheme is relatively small and depends on the average degree. In sparse graphs, the performance of both implementations of Rule k is very close to that of the tree scheme.

Figure 7 compares the performance of the restricted Rule 1, the combination of restricted Rules 1 and 2, and four different implementations of Rule k (restricted, non-restricted, based on 2-hop information, and based on 3-hop information). Both undirected graphs (the upper row) and directed graphs with 10% unidirectional links are simulated. Rule k performs better than the restricted Rule 1 and the combination of restricted Rules 1 and 2. Among the different implementations of Rule k, the non-restricted implementation performs better than the implementation based on 3-hop information, which in turn, performs better than the implementation based on 2-hop information and the restricted implementation. The difference between different implementations of Rule k is

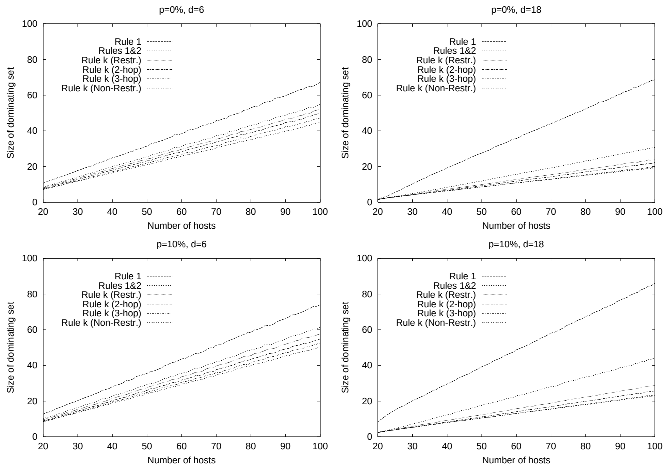

Fig. 7. Different dominant pruning rules.

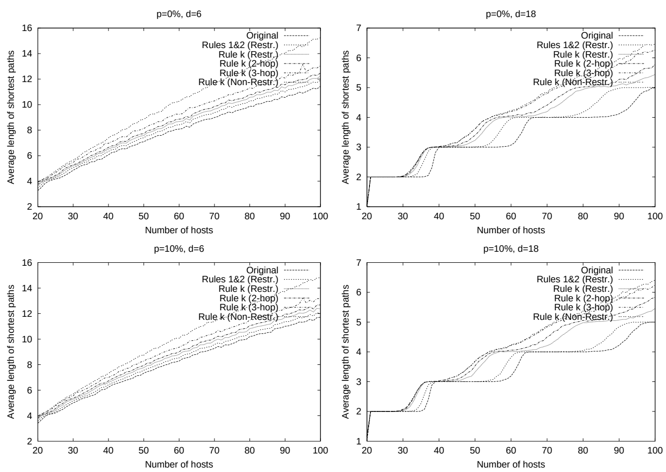

Fig. 8. Average end-to-end distance increment.

relatively small. Therefore, the restricted implementation or the implementation based on 2-hop information is preferable due to lower communication cost. In sparse and undirected graphs, the performance of Rule k is close to that of the combination of restricted Rules 1 and 2 (yet still much better than that of Rule 1 only). In relatively dense and directed graphs, the restricted Rule k performs much better than the combination of restricted Rules 1 and 2.

Figure 8 shows the path length increment caused by dominating-set-based routing. For each randomly generated graph, we compute the all-pair shortest paths with no constraint (*Original*). Then for each dominating set computed by the marking process and different dominant pruning rules, we compute the all-pair shortest paths with the constraint that intermediate vertices are restricted to gateways. The average end to end distance is computed as the average number of hops in these paths. The simulation results show that forwarding data along gateways will not increase the end to end distance significantly. When the restricted Rules 1 and 2 or the restricted Rule k are used to generate the dominating set, the average distance increment is less than 10%. When other versions of Rule k are used, the average distance becomes longer. However, even when the non-restricted Rule k is used, the average distance increment is still within 20%.

Simulation results can be summarized as follows: (1) The connected dominating set produced by the marking process and the restricted Rule k is about the same size as those by the cluster-based schemes and this is achieved in a localized way without sequential propagation, (2) Rule k is a more efficient dominant pruning rule than Rules 1 and 2 and can be implemented without increasing complexity, (3) Rule k outperforms Rules 1 and 2 significantly in networks with relatively high density and/or high percentage of unidirectional links, and (4) forwarding data along gateways will not increase the end-to-end distance significantly.

VI. Conclusions

We have proposed a generic dominant pruning rule called Rule k to further reduce the size of a connected dominating set constructed by Wu and Li's marking process [4]. An efficient algorithm has been proposed to implement Rule k in a "restricted" manner, which is almost as efficient in reducing the dominating set as the "full" version. The restricted Rule k algorithm has less overhead than the combination of two former dominant pruning rules called Rules 1 and 2. Simulation results show that the restricted Rule k outperforms the combination of restricted Rules 1 and 2 in reducing the dominating set. Both marking process and Rule k support unidirectional links. In networks without unidirectional links, the marking process with the restricted Rule k is as efficient as several cluster-based schemes. Furthermore, the restricted Rule k is applied in a pure localized manner with constant rounds of information exchanges. Our future research includes applying the dominant pruning rules to the k-hop dominating set to make dominating-set-based routing more scalable.

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