1. (25%) A $k$-combination of an $n$-set $S$ is simply a $k$-subset of $S$. For example, \{ab, ac, ad, bc, bd, cd\} is the 2-combination of the 4-set \{a, b, c, d\}. Write a pseudocode to generate the 3-combination of an $n$-set $S$. Show the correctness of your code by showing the 3-combination of the 5-set \{a, b, c, d, e\}. What is the complexity of your solution?

2. (25%) Modify the HEAPIFY $(A, i)$ (page 143) for a ternary heap which is a complete ternary tree where three children of a node are called LEFT, CENTER, and RIGHT. Assume that the heap property remains the same.

3. (25%) Give an efficient algorithm to count the total number of paths in a directed acyclic graph (DAG) from $u$ to $v$. Apply your algorithm to Figure 25.8 (page 537) where $u = r$ and $v = x$.

4. (25%) Given a green onion of $n$ inches, you are required to cut it into $n$ pieces of one inch each. Assume that after the onion has been cut into $k$ pieces, they can be “piled” together so that the following cut can generate up to $2k$ pieces. What is the minimum number of cuts? Determine a greedy approach that generates a minimum number of cuts. Show the correctness of your approach for $n = 13$.

(Bonus Points, 5 pts)

5. In the activity-selection problem of $n$ activities ($n$ is a large integer, say $n > 1,000$), suppose a greedy approach always selects a compatible activity with an earliest starting time. Provide a sufficient condition such that this greedy approach always generate an optimal solution.