COT 6401 The Analysis of Algorithms

Homework 1

Due: February 11

All for solutions, provide explanation first in English followed by pseudo code. A brief complexity analysis, including how to derive the result, is also needed.

1. (divide-and-conquer using transform-and-conquer) Compute a mode, where *mode* is a value that occurs most often in a given list of numbers. For example, for 2, 4, 6, 2, 6, 1, 6, the mode is 6. Design an algorithm with complexity $\Theta(n \log n)$.

2. (divide-and-conquer and dynamic programming) Suppose your job at an investment company is to buy x shares of a stock on some day and sell all these shares on some (later) day. There are $i = 1, 2, .., n$ days. The share price at day i is $p(i)$. Design two efficient algorithms that generate the maximum profit by deciding when to buy and sell. The first solution uses divide-and-conquer with complexity $\Theta(n \log n)$. The second solution applies dynamic programming to reduce the complexity to $\Theta(n)$.

3. (greedy algorithm) Coin changing problem 16-1, page 402 in the textbook.

4. (stable marriage problem) Suppose $2n$ people (n men and n women) are either bad or good. Specifically, there are k good men and k good women. In the preference list, everyone would rather marry any good person than any bad person. Show that in a stable marriage, every good man is married to a good woman.

5. (**bonus problem**) Suppose you are given an array A with n entries with distinct values. Assume the values in the array is *unimodel*: For some index p between 1 and n, the values in the array entries increase up to position p and then decrease the remainder of the way until position n. Show how to find the entry p by reading at most $O(\log n)$ entries in A and then how to minimize the total number of readings.