Section I -- Definitions

1. Precisely define the following.
   a. Grammar (10 pts)
   b. Deterministic Finite Accepter (10 pts)
   c. Regular Expression (10 pts)
   d. The language $L(G)$ generated by the grammar $G$. (5 pts)

Section II -- Theory

2. The reverse of a string $w \in \sum^*$, denoted $w^R$, is defined by the recursive rules $a^R = a$ and $(wa)^R = aw^R$ for $a \in \sum$. Using the fact that $(uv)^R = v^R u^R$ for all strings $u, v \in \sum^*$, prove by induction that $(w^R)^R = w$. (15 pts)

3. Use a diagram to describe the construction on an automaton that accepts the language $L^*$ from an automaton that accepts $L$. (5 pts)

4. Prove that all finite languages are regular. (10 pts)

Section III -- Applications

5. Find a grammar that generates the language $L = \{ a^n b^{2n} : n \geq 1 \}$. (5 pts)

6. Consider the language $L = \{ xwx : x \in \{a,b\} \text{ and } w \in \{a,b\}^* \}$. (10 pts)
   a. Describe the language in words.
   b. Give a nfa that accepts $L$.
   c. Give a regular expression for $L$.

7. Describe in words, in the simplest possible way, the language generated by $(a^*b^*)^*$. (5 pts)

8. Consider the language $L$ of strings containing the substring 101 over the alphabet $\{0,1\}$. (15 pts)
   a. Give a dfa that accepts $L$.
   b. Modify the dfa of part a. to give a dfa which accepts $\overline{L}$.
   c. Derive a regular grammar for $\overline{L}$ from the dfa of part b. List only the productions. You may use the short notation $A \rightarrow xB \mid yC$ for multiple productions with the same left-hand side if you wish.
   Note: Partial credit will be given for the proper transformation from a to b and from b to c even if the answer to the previous part is incorrect.